INT. J. CONTROL, 2003, VOL. 76, NO. 7, 643–656

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Sturdy control of discrete communication networks. Part II: Call reshaping

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The problem of optimizing the efficiency of large capacity communication networks is considered. An important objective is to reduce the effects of traffic uncertainties on network efficiency, especially in cases where statistical models of the traffic uncertainties are not available. The main result is the development of algorithms for adjusting the data flow of each network customer so as to optimize network efficiency. It is shown that, in some common situations, the use of feedback control makes it possible to eliminate completely the effects of traffic uncertainties on network efficiency, despite the lack of statistical models.

1. Introduction

Many of the signals commonly transmitted through digital communication networks lack detailed statistical models and incorporate large unmodelled uncertainties. The present paper concentrates on the optimization of network performance in the presence of such signals. By and large, traffic control algorithms can utilize two tactics to help improve network efficiency: (i) selective admission, whereby only the most auspicious calls are allowed into the network; and (ii) reshaping of the signals passing through the network. The reshaping of the signals is accomplished by buffers that store cells temporarily and release them later back into the flow. Traffic control that involves the use of buffering is referred to as *dynamic traffic control*.

The issue of admission control was discussed in detail in the first part of the paper (Hammer 2003). The present part combines admission control with reshaping of call waveforms to achieve maximal network efficiency. The optimization criterion we use is asymptotic efficiency (see Hammer 2003). This criterion aims at the optimization of large capacity networks.

The present paper concentrates on the quantitative aspects of the call reshaping process. Among other issues, we examine how to reshape call waveforms to help reduce the effects of uncertainties on the asymptotic efficiency of the network. In some common cases, appropriate reshaping of the call waveforms may yield an asymptotic backbone efficiency of 1, even when large unmodelled uncertainties are present.

A discrete communication network can be described by figure 1. Here, A represents the gate into the network; P and C are compensators used to control the flow of cells; Σ represents a relatively short network segment that connects the source controller C to the backbone controller P; Ξ represents the backbone; and T represents the destination of the data.

Following Hammer (2003), a call is modelled as a sum

$$c = \chi + v \tag{1}$$

where χ represents the deterministic (or nominal) part of the call, while v represents the uncertain part. Both χ and v are piecewise constant functions over the partition $\{I_1, \dots, I_q\}$ of the call cycle $[1, T]$ (see Hammer 2003, § 2 for more details and notation). The only information available about the uncertain part v is an amplitude bound

$$0 \leq A(v) \leq \rho \tag{2}$$

where $A(v) := \max\{|v_k| \colon k = 1, \dots, T\}$ represents the amplitude of v. No information is provided about the statistics of the uncertain part.

The primary traffic control operations of the network are performed by the compensator P, which is located at the gate of the backbone. The role of the source compensator C is secondary; it provides long term storage, to help reduce the storage requirements of the router compensator P. Due to the subordinate role of C, it can be ignored when discussing the main outlines of the traffic control problem. Remark 1 later indicates that much of the cell storage capacity required for effective buffering can be delegated to C, without adversely affecting backbone performance.

The present paper concentrates on sturdy traffic control, namely, on the lossless control of network traffic under conditions of large and unmodelled uncertainties. The lossless transmission requirement sets sturdy traffic control somewhat apart from the more traditional statistical traffic control algorithms, which do permit cell losses during certain rare traffic events (see, e.g. Atkins 1980, Golestani 1991 and Chang 1994). It is shown in § 5 below that one can often achieve full asymptotic efficiency with no cell losses, despite large call uncertainties.

Received 1 July 2000. Revised 1 August 2002. Accepted 1 February 2003. Communicated by Professor H. Kimura.

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International Journal of Control ISSN 0020–7179 print/ISSN 1366–5820 online © 2003 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/0020717031000102970

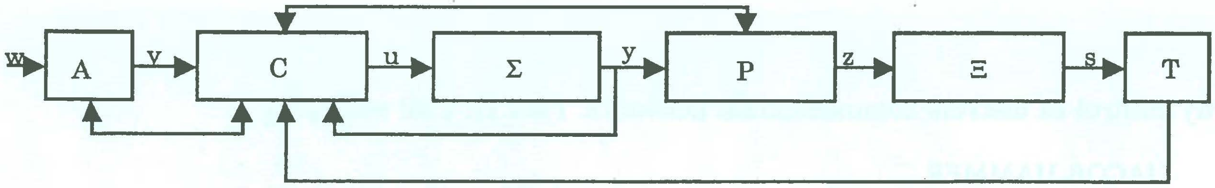

Figure 1.

The use of traffic control to reshape call waveforms entails, of course, the delay some of the cells passing through the network. Such cell delays are referred to as *buffering delays*. The service specifications of each call class dictate the maximal buffering delay (or jitter) a cell of the class may incur. Let $\{c^1, \ldots, c^m\}$ be the family of call classes associated with our network, and let $\tau(r)$ be the maximal buffering delay permitted for a cell of the class c^r, $r = 1, \ldots, m$. We assume throughout our discussion that all cells can be delayed by at least one step, i.e. that

$$\tau(r) \geq 1, \qquad r = 1, \ldots, m$$

In general, certain call classes permit more buffering delay than others. For example, the class of computer data file transfers allows substantial buffering delays, while classes such as streaming audio or streaming video have a relatively low tolerance of buffering delays and jitter. For the extreme case where a class c^r has no cell delay restriction, we set $\tau(r) := T - 1$. The traffic control algorithms developed in this paper abide by all buffering delay constraints. In fact, maximal buffering delay is the only service requirement taken into consideration in our present discussion, as it is the one that most critically affects dynamic traffic control. Other service requirements are considered in ATM Forum (1997), Handel *et al.* (1994) and Schwartz (1996).

The network optimization process developed in this paper is a global process that relies on the available call descriptions. It is not based solely on the momentary traffic load of the network, as are some of the classical traffic control algorithms (e.g. Decina and Toniatti 1990, Rathgeb 1991, CCITT 1992).

The present study is a continuation of Hammer (2003) and depends on the notation and on the results introduced there. It is organized as follows. Section 2 includes the basic framework for dynamic control of discrete communication networks. The shuffling algorithm, which forms an important tool in later sections of the paper, is described in § 3. In § 4 we discuss the process of reshaping call waveforms for improving network efficiency. The paper concludes with § 5, where we examine the use of dynamic compensation to reduce the effects of call uncertainties on network efficiency. The results of § 5 indicate that, in some common situations, the effects of call uncertainties on network efficiency can be completely eliminated, even in cases where statistical models of the uncertainties are not available.

2. Reshaping of deterministic calls

To clarify the basic issues involved with dynamic traffic control, we start by considering the case of deterministic calls, namely, of calls of the form (1) with $v = 0$. For convenience, here are a few items from Hammer (2003):

(i) A family of call classes $\{c^1, \ldots, c^m\}$ is complete if there are integers $\alpha_1, \ldots, \alpha_m \geq 0$ such that $\sum_{i=1}^m \alpha_i c^i$ is a non-zero constant over $[1, T]$.

(ii) A flow z through a backbone of capacity ϕ is *lossless* if its amplitude satisfies $A(z) \leq \phi$.

(iii) c_k^i denotes the value of a call c^i at a step k.

The next statement demonstrates that an incomplete family of call classes can be transformed into a complete family by adding one more call class. The statement also characterizes the minimal fraction of backbone capacity that has to be devoted to the new class to achieve asymptotic efficiency of 1.

For a family $F = \{c^1, \ldots, c^m\}$ of call classes and an additional call class c, denote by $F(c) := \{c^1, \ldots, c^m, c\}$ the augmented family of calls. Consider a lossless flow $z(\phi) = \sum_{i=1}^m \alpha_i(\phi)c^i + \alpha_{m+1}(\phi)c$ through a backbone of capacity ϕ; here, $\alpha_1(\phi), \ldots, \alpha_{m+1}(\phi) \geq 0$ are integers. The *capacity-fraction* devoted to the call c is defined by

$$\eta(c, z(\phi)) := \frac{\sum_{k=1}^T \alpha_{m+1}(\phi)c_k}{T\phi}$$

Finally, recall that the maximal asymptotic efficiency η^* of the family F was calculated in Hammer (2003, Theorem 3).

Proposition 1: *Let $F = \{c^1, \ldots, c^m\}$ be an incomplete family of call classes over the partition $\{I_1, \ldots, I_q\}$ of $[1, T]$, let η^* be the maximal asymptotic efficiency of the family F, and let ϕ denote the backbone capacity:*

(i) *There is a class class c over the partition $\{I_1, \ldots, I_q\}$ for which the augmented family $F(c)$ is complete.*

(ii) *Let c be any call class over the partition $\{I_1, \ldots, I_q\}$ for which $F(c)$ is a complete family,* *and let $z(\phi)$ be any lossless flow of the family $F(c)$ with asymptotic efficiency of 1. Then, the capacity-fraction devoted to the class c satisfies* $\lim_{\phi\to\infty} \eta(c, z(\phi)) \geq 1 - \eta^*$.

(iii) *There is a call class c^+ over the partition $\{I_1, \ldots, I_q\}$ such that $F(c^+)$ is a complete family and the following holds. There is a lossless flow $z^+(\phi)$ of the family $F(c^+)$ with asymptotic efficiency of* 1 *and with capacity-fraction devoted to the class c^+ satisfying* $\lim_{\phi\to\infty} \eta(c^+, z^+(\phi)) = 1 - \eta^*$.

The proof of Proposition 1 is in the Appendix. In view of Proposition 1, one can always construct a call class that completes a given family F into a complete family of call classes. The new class can be selected so that it lifts the asymptotic backbone efficiency to one, without affecting the transmission efficiency of the original family F.

In practice, it is not always possible to add a new call class. It is usually more practical to reshape the waveforms of the family's existing members, in an attempt to obtain a complete family. Waveforms of call classes can be reshaped by storing cells in buffers, and releasing the stored cells during later steps of the calls. Of course, such reshaping must be performed without violating the maximal delay specifications of the affected calls. We discuss next a somewhat restricted version of this problem, where only one of the call classes is being reshaped.

Consider an incomplete family $F = \{c^1, \ldots, c^{m+1}\}$ of call classes. We examine the conditions under which the call class c^{m+1} can be reshaped into a call class c for which the family $F(c) := \{c^1, \ldots, c^m, c\}$ is complete. In this process, a number of calls of the class c^{m+1} may be transformed into a (possibly different) number of calls of the class c. For instance, γ_1 calls of c^{m+1} may be transformed into γ_2 calls of c, where γ_1, γ_2 are positive integers. Note that changing the waveform of a call does not hamper its content as long as the service requirements are met.

Our call reshaping process must abide by two constraints:

(i) Causality—cells can only be delayed (not advanced) in time; and

(ii) Service requirements—cells cannot be delayed longer than the maximal delay specification of the class.

Initially, we shall also require the following:

(iii) no cells can be delayed beyond the time cycle time T.

Referring to figure 1, constraint (i) means that the number of cells that have exited the compensator P up to a step k cannot exceed the number of cells that have entered P up to the step k. Constraint (iii) means that all call processing is confined to the interval $[1, T]$, namely, that the number of cells entering P during the interval $[1, T]$ must equal the number of exiting cells during this interval. In formal terms, constraints (i) and (iii) reduce to the following.

Lemma 1: *Let c' and c'' be two non-empty call classes over the interval $[1, T]$. Assume that c' has no cell delay restriction. Then, the following two statements are equivalent:*

(i) *Every γ_1 calls of the class c' can be transformed into γ_2 calls of the class c'' with no cell loss.*

(ii) $\gamma_1 \sum_{i=1}^{k} c'_i \geq \gamma_2 \sum_{i=1}^{k} c''_i, \quad k = 1, \ldots, T,$ *with equality for $k = T$.*

The integers γ_1 and γ_2 of Lemma 1 can be characterized easily in terms of the total number of cells contained in the calls c' and c''. Indeed, for a call c, let $\#c$ be the total number of cells contained in the call. When transforming γ_1 calls of c' into γ_2 calls of c'' without cell loss, we must clearly have $\gamma_1\#c' = \gamma_2\#c''$. Now, let $\ell > 0$ be a least common integer multiple of $\#c'$ and $\#c''$, and let $\gamma', \gamma'' > 0$ be integers satisfying

$$\ell = \gamma'\#c' = \gamma''\#c'' \tag{3}$$

By basic properties of least common multiples, there then is an integer $\gamma > 0$ such that

$$\gamma_1 = \gamma\gamma' \quad \text{and} \quad \gamma_2 = \gamma\gamma'' \tag{4}$$

Equation 4 indicates that when discussing transformations of calls of the class c' into calls of the class c'', we can restrict our attention to the conversion of γ' calls of c' into γ'' calls of c''. It is important to note that γ' and γ'' are uniquely determined by the call classes c' and c''.

To simplify notation, define the new call classes $\omega := \gamma_1 c'$ and $\Omega := \gamma_2 c''$, so that one call ω is transformed into one call Ω. In general, there can be several incoming classes, say $\omega^1, \ldots, \omega^m$. The sum

$$z := \omega^1 + \cdots + \omega^m \tag{5}$$

may be combined and transformed in the compensator P into a single outgoing class Ω over the call cycle $[1, T]$. Here, z is regarded as the incoming call and Ω as the outgoing call. Then, Lemma 1 takes the following form.

Lemma 2: *Let $\omega^1, \ldots, \omega^m$ and Ω be non-empty call classes over the interval $[1, T]$. Assume that the classes*

$\omega^1, \ldots, \omega^m$ have no cell delay restrictions. Then, the following two statements are equivalent:

(i) *The combination $z := \omega^1 + \cdots \omega^m$ can be transformed into a call of the class Ω with no cell loss.*

(ii) *$\sum_{i=1}^{k} z_i \geq \sum_{i=1}^{k} \Omega_i$, $k = 1, \ldots, T$ with equality for $k = T$.* □

For classes defined over the partition $\{I_1, \ldots, I_q\}$ (where a segment I_i consists of λ_i steps), condition (ii) of Lemma 2 can be rewritten in the equivalent form

$$\sum_{i=1}^{j} \lambda_i z(i) \geq \sum_{i=1}^{j} \lambda_i \Omega(i), \qquad j = 1, \ldots, q \tag{6}$$

with equality for $j = q$.

To take into consideration restrictions on cell delays, let $\tau(r) \geq 1$ be the maximal buffering delay (in number of steps) allowed for cells of the call class ω^r. Clearly, for $k > \tau(r)$, all cells of the class ω^r that have entered the compensator P during the steps $1, \ldots, k - \tau(r)$, must have exited P by the end of step k. Hence, the total number of cells of the call ω^r that must have exited P by the end of step k is given by $\sum_{i=1}^{k-\tau(r)} \omega_i^r$.

To simplify notation, it is convenient to adopt the somewhat unusual convention that

$$\sum_{i=\alpha}^{\beta} a_i := 0 \qquad \text{whenever } \beta < \alpha$$

Then, the class z can be transformed into Ω, while upholding the maximal delay constraint of all classes $\omega^1, \ldots, \omega^m$, if and only if

$$\left.\begin{aligned} &\sum_{r=1}^{m} \sum_{i=1}^{k-\tau(r)} \omega_i^r \leq \sum_{i=1}^{k} \Omega_i, \quad k = 1, \ldots, T-1 \\ &\sum_{r=1}^{m} \sum_{i=1}^{T} \omega_i^r = \sum_{i=1}^{T} \Omega_i \end{aligned}\right\} \tag{7}$$

Combining with Lemma 2, this leads to the following statement, which characterizes the conditions for waveform transformations.

Proposition 2: *Let $\omega^1, \ldots, \omega^m$ and Ω be non-empty call classes over the interval $[1, T]$. Let $\tau(r)$ be the maximal delay permissible for a cell of the class ω^r, $r = 1, \ldots, m$. Then, statements* (i) *and* (ii) *are equivalent:*

(i) *The combination $z := \omega^1 + \cdots + \omega^m$ can be transformed into the class Ω without cell loss.*

(ii) (a) *$\sum_{i=1}^{k} z_i \geq \sum_{i=1}^{k} \Omega_i$, $k = 1, \ldots, T$, with equality for $k = T$, and*

(b) *$\sum_{i=1}^{m} \sum_{i=1}^{k-\tau(r)} \omega_i^r \leq \sum_{i=1}^{k} \Omega_i$, $k = 1, \ldots, T - 1$.*

3. The shuffling algorithm

In the present section we develop an algorithm that implements the waveform transformation of Proposition 2. The algorithm is designed to delay each cell by the minimal possible number of steps. Recall that waveform transformations are performed by using the buffers of the compensator P of figure 1.

Referring to Proposition 2, let ω_i^r be the number of cells of the class ω^r that enter the compensator P during step i. For an integer $j \geq i$, let $c_0(i,j;r)$ be the total number of cells of the set ω_i^r that have left the compensator P by the end of step j, i.e. during the steps $i, i+1, \ldots, j$. Clearly, cells cannot be moved out of P before they have arrived at P, so

$$c_0(i,j;r) := 0 \quad \text{for all } j < i, \quad r = 1, \ldots, m \tag{8}$$

Let $\gamma(i,j;r)$ denote the number of cells of ω_i^r that leave P during step j, so that

$$c_0(i,j;r) := c_0(i,j-1;r) + \gamma(i,j;r),$$
$$i,j = 1, \ldots, T, \quad r = 1, \ldots, m$$

Consistency with (8) requires $\gamma(i,j;r) := 0$ for all $j < i$, $r = 1, \ldots, m$. Clearly, a traffic control algorithm is determined by specifying the values of $\gamma(i,j;r)$ for all $i,j \geq i$, and r.

Now, for a step $j \in \{1, \ldots, T\}$, define the set $s(r,j) := \{i \leq j \colon \omega_i^r > c_0(i,j;r)\}$ and the integer

$$a(r,j) := \begin{cases} \{\min i \colon i \in s(r,j)\} & \text{if } s(r,j) \neq \emptyset \\ j & \text{if } s(r,j) = \emptyset \end{cases}$$

When $s(r,j) \neq \emptyset$, then $a(r,j)$ is the earliest step of ω^r from which cells are still stored in P at the end of step j. Consequently, at the step j, the longest delay of a cell of the class ω^r in the buffer of P is $[j - a(r,j)]$ steps. The maximal additional delay that such a cell can tolerate is then

$$b(r,j) := \tau(r) - [j - a(r,j)] \tag{9}$$

We call $b(r,j)$ the *delay reserve* of class ω^r at the step j. Clearly, the maximal cell delay specifications are met if and only if $b(r,j) \geq 0$ for all $j = 1, \ldots, T$ and $r = 1, \ldots, m$.

Now, fix a step $j \in \{1, \ldots, T\}$. Let $\{\pi(1,j), \ldots, \pi(m,j)\}$ be a permutation of the list $\{1, \ldots, m\}$ such that, at the step j, the class $\omega^{\pi(1,j)}$ has the lowest delay reserve; $\omega^{\pi(2,j)}$ has the next lowest delay reserve, and so on, with $\omega^{\pi(m,j)}$ having the highest delay reserve. In other words, $b(\pi(r,j),j) \geq b(\pi(s,j),j)$ for all $r > s$, where $r, s \in \{1, \ldots, m\}$. The number of cells from step i of the class $\omega^{\pi(r,j)}$ left in the buffers of P at the step j is given by $[\omega_i^{\pi(r,j)} - c_0(i,j-1;\pi(r,j))]$.

Recall that Ω is the outgoing call class of Proposition 2. For a step $j \in \{1, \ldots, T\}$, define recursively the following quantities for $r = 1, \ldots, m$ and $i = 1, \ldots, j$

$$\gamma(i,j;\pi(r,j)) := \min\left\{[\omega_i^{\pi(r,j)} - c_0(i,j-1;\pi(r,j))], \left[\Omega_j - \sum_{d=1}^{r-1}\sum_{k=1}^{j}\gamma(k,j;\pi(d,j))\right]\right\} \quad (10)$$

Note that, if $\gamma(k,j;\pi(d,j))$ cells of step k of the class $\omega^{\pi(d,j)}$ are leaving the buffers of P during step j for each $k = 1, \ldots, j$ and $d = 1, \ldots, r-1$, then the maximal number of additional cells that can leave the buffers of P during step j is $\Omega_j - \sum_{d=1}^{r-1}\sum_{k=1}^{j}\gamma(k,j;\pi(d,j))$. In view of the previous paragraph, this implies that $\gamma(i,j;\pi(r,j))$ is the maximal number of cells of step i of $\omega^{\pi(r,j)}$ that can leave the buffers of P at the step j. This leads to the following traffic control algorithm.

The shuffling algorithm: At each step $j = 1, \ldots, T$, inject into the backbone $\gamma(i,j;p)$ cells from step i of the call class ω^p, where $i = 1, \ldots, j$, $p = 1, \ldots, m$, and $\gamma(i,j;p)$ is given by (10).

An examination of the shuffling algorithm shows that it continuously shuffles the population of the cells stored in the buffer of p. It releases into the backbone cells that are closest to their delay limit, while storing in the buffer the newest arrivals. In the special case where all classes have the same maximal delay specification τ, i.e. when $\tau(r) = \tau$ for all $r = 1, \ldots, m$, the shuffling algorithm reduces to serial buffering. The next statement indicates that the shuffling algorithm can be used to transform the incoming combination z of (5) into the outgoing call Ω in all cases in which such a transformation is at all possible.

Theorem 1: *Let $\omega^1, \ldots, \omega^m$ and Ω be non-empty call classes over the interval $[1, T]$, and let $\tau(r)$ be the maximal delay allowed for cells of the class ω^r, $r = 1, \ldots, m$. Then, the following two statements are equivalent:*

(i) *The combination $z := \omega^1 + \cdots + \omega^m$ can be transformed into the class Ω without violating any maximal cell delay specification and without cell loss.*

(ii) *The shuffling algorithm transforms z into Ω without violating any maximal delay specification and without cell loss.*

Proof: Clearly, when (ii) is valid, so is (i). Conversely, assume that (i) is valid. Then, the conditions of Lemma 2(ii) are valid. In view of (10), this implies that $\Omega_j = \sum_{d=1}^{m}\sum_{k=1}^{j}\gamma(k,j;d)$ for all $j = 1, \ldots, T$, so no cell loss occurs. Also, (7), (10), and the fact that $b(\pi(r,j),j) \geq b(\pi(s,j),j)$ whenever $r > s$, imply that $b(r,j) \geq 0$ for all $j = 1, \ldots, T$ and $r = 1, \ldots, m$. Thus, the shuffling algorithm transforms z into Ω without violating any maximal delay requirements, and our proof concludes. □

4. Optimal call transformations

We turn now to the general problem of optimizing the transmission of a family of call classes by using call transformations. For the sake of simplicity, we continue to restrict ourselves to the case where only a single call class is being transformed. The results can be generalized to situations where several call classes are transformed simultaneously.

Let $F = \{c^1, \ldots, c^{m+1}\}$ be a family of call classes over the partition $\{I_1, \ldots, I_q\}$ flowing through a backbone of capacity ϕ; let λ_i be the number of steps of the segment I_i. Assume that a number of copies of the class c^{m+1} is transformed into a class c' over the same partition $\{I_1, \ldots, I_q\}$. Consider the case where α_i calls of the class c^i, $i = 1, \ldots, m$, and α calls of the class c' are injected into the backbone. To simplify notation, it is convenient to introduce the call class $c := \alpha c'$, and consider the family $F(c) := \{c^1, \ldots, c^m, c\}$. The flow through the backbone is then given by

$$z := \sum_{i=1}^{m} \alpha_i c^i + c \quad (11)$$

The amplitude of this flow is $A(z)$. To simplify the terminology, we say that the class c^{m+1} is *transformed* into the class c, although several copies of c^{m+1} might have been used in this process.

Define the rational numbers $\beta_i := \alpha_i / A(z)$, $i = 1, \ldots, m$. Recalling that $c(j)$ is the value of c on the segment I_j, define the rational waveform

$$\theta'(j) := c(j)/A(z), \qquad j = 1, \ldots, q$$

Taking into account the fact that all quantities are non-negative, we obtain the relations

$$0 \leq \theta'(j) \leq 1, \qquad j = 1, \ldots, q$$

$$\sum_{i=1}^{m} \beta_i c^i(j) + \theta'(j) = \frac{z(j)}{A(z)} \leq 1, \qquad j = 1, \ldots, q$$

In Hammer (2003, equation (31)) we defined the quantities

$$\psi_i = \frac{1}{T}\sum_{j=1}^{q} \lambda_j c^i(j), \qquad i = 1, \ldots, m$$

In terms of these quantities, the relative efficiency of the flow z is

$$\eta_r(\alpha_1, \ldots, \alpha_m, c) = \frac{\sum_{k=1}^{T} z_k}{TA(z)} = \sum_{i=1}^{m} \beta_i \psi_i + \frac{1}{T}\sum_{j=1}^{q} \lambda_j \theta'(j) \quad (12)$$

Working directly with θ' is rather complicated, due to the dependence of θ' on the amplitude $A(z)$ of the combined flow z. To circumvent this difficulty, note that, in order to achieve maximal efficiency as the backbone capacity ϕ approaches infinity, the amplitude $A(z)$ must satisfy

$$\lim_{\phi\to\infty} A(z)/\phi = 1 \tag{13}$$

Thus, when considering the case $\phi \to \infty$, one can replace $A(z)$ by ϕ. If so, one can replace $\theta' = c/A(z)$ by

$$\theta := \frac{c}{\phi} = \frac{A(z)}{\phi}\theta' \tag{14}$$

as $\phi \to \infty$. The advantage of doing so is the fact that θ bears a linear relationship to c, while θ' does not.

Now, recall that the class c is obtained by a transformation of the class c^{m+1}. Let $\ell > 0$ be a least common multiple of integers $\#c^{m+1}$ and $\#c$. In analogy to (3), define the integers $\gamma' := \ell/\#c^{m+1}$ and $\gamma'' := \ell/\#c$. Combining the γ' copies of c into one call, we redefine the class c so that $\gamma'' = 1$. Then, using Lemma 1, (4), and (6), we can write

$$\left.\begin{aligned} &\gamma'\sum_{i=1}^{j}\lambda_i c^{m+1}(i) \geq \sum_{i=1}^{j}\lambda_i c(i), \qquad j = 1,\dots,q-1\\ &\gamma'\sum_{i=1}^{q}\lambda_i c^{m+1}(i) = \sum_{i=1}^{q}\lambda_i c(i)\end{aligned}\right\} \tag{15}$$

Next, let $\tau \geq 0$ be the maximal cell delay specification for the class c^{m+1}. We can rewrite (7) in the form

$$\gamma'\sum_{j=1}^{k} c_j^{m+1} \leq \sum_{j=1}^{k+\tau} c_j, \qquad k = 1,\dots,T-\tau \tag{16}$$

Dividing the relations (15) and (16) by the backbone capacity ϕ, yields

$$\begin{aligned} &\frac{\gamma'}{\phi}\sum_{i=1}^{j}\lambda_i c^{m+1}(i) \geq \frac{1}{\phi}\sum_{i=1}^{j}\lambda_i c(i) \qquad j = 1,\dots,q-1\\ &\frac{\gamma'}{\phi}\sum_{i=1}^{q}\lambda_i c^{m+1}(i) = \frac{1}{\phi}\sum_{i=1}^{q}\lambda_i c(i)\\ &\frac{\gamma'}{\phi}\sum_{j=1}^{k} c_j^{m+1} \leq \frac{1}{\phi}\sum_{j=1}^{k+\tau} c_j, \qquad k = 1,\dots,T-\tau\end{aligned}$$

Finally, using (14), defining $\beta_{m+1} := \gamma'/\phi$, and employing the call pool parameter ρ_{m+1} of the class c^{m+1} (Hammer 2003, § 6), we obtain the relations

$$\left.\begin{aligned} &\beta_{m+1}\sum_{i=1}^{j}\lambda_i c^{m+1}(i) - \sum_{i=1}^{j}\lambda_i\theta(i) \geq 0, \qquad j = 1,\dots,q-1\\ &\beta_{m+1}\sum_{i=1}^{q}\lambda_i c^{m+1}(i) - \sum_{i=1}^{q}\lambda_i\theta(i) = 0\\ &\sum_{j=1}^{k+\tau}\theta_j - \beta_{m+1}\sum_{j=1}^{k} c_j^{m+1} \geq 0 \quad k = 1,\dots,T-\tau\\ &\beta_{m+1} \leq \rho_{m+1}\\ &\beta_{m+1} \geq 0\end{aligned}\right\} \tag{17}$$

Reversing the point of view, we can use the relations (17) as the basis of an optimization process yielding a rational flow θ that maximizes the asymptotic efficiency of the backbone. To this end, define the linear function

$$L := \sum_{j=1}^{m}\beta_j\psi_j + \frac{1}{T}\sum_{i=1}^{q}\lambda_i\theta(i) \tag{18}$$

where $\beta_1,\dots,\beta_m$ and $\theta(1),\dots,\theta(q)$ are regarded as variables, while $\psi_1,\dots,\psi_m$, $\lambda_1,\dots,\lambda_m$, and T are fixed parameters of the specified family F, the partition $\{I_1,\dots,I_q\}$, and the call cycle T. In view of (12), (13), and (14), the function L represents the relative efficiency of the flow in the limit, as $\phi \to \infty$. Consider then the maximization of L under the linear constraints

$$\left.\begin{aligned} &\text{(i)}\ \sum_{i=1}^{m}\beta_i c^i(j) + \theta(j) \leq 1, \qquad j = 1,\dots,q\\ &\text{(ii)}\ \theta(j) \leq 1, \qquad j = 1,\dots,q\\ &\text{(iii)}\ \theta(j) \geq 0, \qquad j = 1,\dots,q\\ &\text{(iv)}\ \beta_i \geq 0, \qquad i = 1,\dots,m+1\\ &\text{(v)}\ \beta_i \leq \rho_i, \qquad i = 1,\dots,m+1\\ &\text{(vi)}\ \beta_{m+1}\sum_{i=1}^{j}\lambda_i c^{m+1}(i) - \sum_{i=1}^{j}\lambda_i\theta(i) \geq 0,\\ &\qquad\qquad j = 1,\dots,q-1\\ &\text{(vii)}\ \beta_{m+1}\sum_{i=1}^{q}\lambda_i c^{m+1}(i) - \sum_{i=1}^{q}\lambda_i\theta(i) = 0\\ &\text{(viii)}\ \sum_{j=1}^{k+\tau}\theta_j - \beta_{m+1}\sum_{j=1}^{k} c_j^{m+1} \geq 0,\\ &\qquad\qquad k = 1,\dots,T-\tau\end{aligned}\right\} \tag{19}$$

Arguments similar to the ones used in the proof of Hammer (2003, Theorem 4) yield the following result regarding the optimization of the backbone flow by reshaping one call class.

Theorem 2: *Let $F = \{c^1, \ldots, c^m, c^{m+1}\}$ be an incomplete family of call classes over the partition $\{I_1, \ldots, I_q\}$ of the interval $[1, T]$, and let $\tau \leq T - 1$ be the maximal delay permitted for the call class c^{m+1}. Let ϕ be the backbone capacity, and let $\rho_1, \ldots, \rho_{m+1}$ be the call pool parameters. Assume that the call class c^{m+1} is transformed into a call class c, and let $F(c) := \{c^1, \ldots, c^m, c\}$ be the resulting family. Let $\eta(\phi, c)$ be the maximal backbone efficiency achievable for the family $F(c)$ under the specified call pool constraints. Denote by L^* the maximal value of the function L of* (18) *under the constraints* (19). *Then, the following are true:*

(i) $\eta(\phi, c) \leq L^*$; *and*
(ii) *there is a class c^* for which* $\lim_{\phi \to \infty} \eta(\phi, c^*) = L^*$.

The class c^* which yields the maximal asymptotic backbone efficiency of Theorem 2 can be obtained as follows. Let $\theta^*(1), \ldots, \theta^*(q)$, $\beta_1^*, \ldots, \beta_{m+1}^*$ be values at which the maximum of L occurs under the constraints (19). Arguments similar to the ones employed in the proof of Hammer (2003, Theorem 4) imply that $\theta^*(1), \ldots, \theta^*(q)$, $\beta_1^*, \ldots, \beta_{m+1}^*$ are rational numbers. Let $a > 0$ be a least common integer denominator of $\theta^*(1), \ldots, \theta^*(q)$, $\beta_1^*, \ldots, \beta_{m+1}^*$. Then, the discussion leading to Theorem 2 shows that an appropriate class c^* is given by

$$c^* := a\theta^*$$

The same discussion also shows that the non-negative integer

$$\gamma^* := a\beta_{m+1}^*$$

is equal to the number of calls of the class c^{m+1} that are used to build one copy of the class c^*. The integers

$$\alpha_i^* := a\beta_i^*, \qquad i = 1, \ldots, m$$

are the respective populations of the call classes $c^1, \ldots, c^m$ in the optimal call package

$$z^* = \sum_{i=1}^{m} \alpha_i^* c^i + c^*$$

Maximal asymptotic efficiency is then achieved by a flow formed by integer multiples of z^*. For a given backbone capacity ϕ, an integer β is selected through the integer division algorithm $\phi = \beta A(z^*) + r$, where $0 \leq r < A(z^*)$. The traffic control algorithm that achieves maximal asymptotic efficiency is then as follows:

(i) use the shuffling algorithm to transform γ^* calls of the class c^{m+1} into one call of the class c^*;
(ii) assemble the package z^*; and
(iii) transmit β copies of the package z^* into the backbone.

To corroborate this construction, note that constraints (19vi and vii) guarantee that c^* can be obtained by a causal transformation of γ^* calls of the class c^{m+1}. Constraint (19viii) guarantees that the maximal cell delay specification τ of the class c^{m+1} is not violated during this transformation.

The traffic control algorithm induced by Theorem 2 provides an optimal flow control strategy in cases where only one of the call classes passing through the network (i.e. c^{m+1}) permits buffering delays. If several of the call classes passing through the network permit buffering delays, Theorem 2 can be generalized as follows. First, regard the class c of (11) as a linear combination (with indeterminate coefficients) of all members of F that permit buffering delays. The optimal coefficients of this linear combination can then be obtained through the solution of a linear programming problem, in close analogy to the approach used in Theorem 2. The resulting coefficients will be rational numbers, and a basic call package is assembled in analogy to the basic call package z^* above. Finally, the integer β is calculated for the given backbone capacity ϕ. This results in a traffic control algorithm that achieves maximal asymptotic backbone efficiency. Note that no generality is lost by considering a single output class c^*, since all output classes can be combined into one class.

5. Dynamic traffic control for calls with uncertainties

We turn now to the development of sturdy traffic control algorithms for families of call classes with amplitude uncertainties. Sturdy traffic control allows us to reduce—or sometimes even completely eliminate—the effects of call uncertainties on backbone efficiency, without employing statistical models of the uncertainties. The traffic control algorithms discussed in the present section depend on the use of feedback control. The use of feedback has been a mainstay of communication networks from the early days (e.g. ATM Forum 1994, 1996, Ramakrishnan and Jain 1988 and Bolot and Shankar 1990). The present discussion continues this tradition by incorporating the principles of sturdy control.

We examine here the effects of uncertainties on families of calls whose nominal parts form a complete family. We show that, in many common cases, it is possible to achieve asymptotic efficiency of 1, despite the uncertainties. We start with the simplest case: families of call classes whose maximal permissible cell delay exceeds the call cycle length T. Such call classes include short duration calls, like downloads of web pages; calls that consist of separate short segments, like phone calls; computer file transfers; and other similar call classes.

Consider then a family of call classes $F = \{c^1, \ldots, c^m\}$ over the partition $I = \{I_1, \ldots, I_q\}$ of the interval $[1, T]$. Following (1) and (2), each call is given by a sum $c^i = \chi^i + \upsilon^i$, $i = 1, \ldots, m$, where χ^i is the deterministic part, υ^i the uncertain part, and $0 \le \upsilon^i \le \rho$. The values of υ^i may vary from one sample of the call c^i to another, and are not known in advance.

In the special case where none of the classes of the family F imposes any restrictions on cell delays, backbone efficiency can be improved as follows. Transmit initially through the backbone a number of cells equal to the number of cells contained in deterministic parts $\chi^1, \ldots, \chi^m$ of the calls; store the remaining cells in the buffer system. Then, the total number of cells stored for the call c^i is

$$\sigma^i = \sum_{t=1}^{T} \upsilon_t^i \tag{20}$$

Note that although σ^i is *a priori* a random number, its value becomes known at the end of the call c^i, i.e. at the step T. In somewhat oversimplified terms, if the cell delay restriction of the class c^i permits, then the σ^i cells can be transmitted as a constant call of amplitude σ^i/T during the second call cycle $[T+1, 2T]$. This will completely eliminate the effect of the uncertainties on backbone efficiency.

As an example, consider the special case where the family $F(\chi) = \{\chi^1, \ldots, \chi^m\}$ of the deterministic parts forms a complete family of call classes. Let $\alpha_1, \ldots, \alpha_m \ge 0$ be integers for which the combination

$$z = \sum_{i=1}^{m} \alpha_i \chi^i \tag{21}$$

is a constant non-zero valued function over the interval $[1, T]$. The corresponding combination of incoming call classes is then

$$z(c) := \sum_{i=1}^{m} \alpha_i c^i$$

Assume that $\beta > 0$ copies of $z(c)$ have been admitted into the compensator P, so that a total of $\beta\alpha_i$ samples of the call c^i are being processed, $i = 1, \ldots, m$. As each call sample may have a different uncertain part, label the samples of the call c^i by $c^{i,j} = \chi^i + \upsilon^{i,j}$, $j = 1, \ldots, \beta\alpha_i$, where $0 \le \upsilon^{i,j} \le \rho$. Then, as in (20)

$$\sigma^{i,j} := \sum_{t=1}^{T} \upsilon_t^{i,j}$$

is the number of cells of the call $c^{i,j}$ stored at the end of the call cycle. The bound ρ on the amplitude of the uncertain parts implies that $0 \le \sigma^{i,j} \le T\rho$. Let $\sigma(i, \beta\alpha_i)$ be the total number of cells of calls of the class c^i that are stored at the end of the step T. Then,

$$\sigma(i, \beta\alpha_i) = \sum_{j=1}^{\beta\alpha_i} \sigma^{i,j} \le \beta\alpha_i T\rho, \qquad i = 1, \ldots, m$$

The total number of cells $\sigma(\beta)$ stored at the end of step T for the β copies of $z(c)$ is then

$$\sigma(\beta) := \sum_{i=1}^{m} \sigma(i, \beta\alpha_i) \le \beta T\rho\left(\sum_{i=1}^{m} \alpha_i\right) =: \delta(\beta) \tag{22}$$

The value of $\sigma(\beta)$ becomes precisely known at the end of the step T. At that point, the $\sigma(\beta)$ stored cells can be shaped into a desirable waveform and transmitted over the next call period, if cell delay restrictions permit. Note that not all $\sigma(\beta)$ cells must be stored within the buffers of P; most of these cells can usually be stored within their source buffers, as explained in Remark 1 below.

One option is to transmit the $\sigma(\beta)$ stored cells into the backbone spread as evenly as possible over next call period $[T+1, 2T]$. This results in a call of amplitude

$$\ell(\beta) = [\sigma(\beta)/T]^+$$

where $[\cdot]^+$ denotes the least integer not smaller than $[\cdot]$. Clearly, $|\ell(\beta) - [\sigma(\beta)/T]| \le 1$ and, when the uncertainties are not all zero, $\sigma(\beta) \to \infty$ as $\beta \to \infty$. These facts imply that

$$\lim_{\beta\to\infty} \ell(\beta)/[\sigma(\beta)/T] = 1$$

Consequently, this method of transmitting the stored cells achieves a relative efficiency of 1 for large backbones.

Now, let ϕ be the backbone capacity, let z be the constant flow of (21), and assume there are no call pool restrictions. Let β be the largest integer for which $\beta z \le \phi$. In order for the present traffic control scheme to work, we need $\ell(\beta) \le \phi$, so assume that this is the case. Let $\gamma \ge 0$ be the largest integer satisfying $\ell(\beta) + \gamma z \le \phi$. Then, overall asymptotic efficiency of 1 can be achieved as follows. At the start of the first call cycle, admit β copies of $z(c)$. During the first call cycle, transmit the constant flow βz, which equals the number of cells contained in the deterministic parts of the admitted calls; store the remaining cells (which correspond to the uncertain parts of the calls). At the start of the second call cycle, admit γ copies of $z(c)$. During the second call cycle, transmit the constant flow γz (equal to the number of cells contained in the deterministic parts) together with a flow of amplitude $\ell(\beta)$ of cells stored during the previous call cycle. For the new calls, store the number of cells exceeding γz. Continue similarly during subsequent call cycles. A slight reflection shows that this flow scheme achieves asymptotic efficiency of 1, completely eliminating the effects of the uncertainties (assuming the proper conditions hold).

To reduce cell delays, one may transmit the $\sigma(\beta)$ stored cells during the first segment I_1 of the next call cycle, rather than distributing them over the entire interval $[T+1, 2T]$. In such case, new calls must be admitted into the network at the beginning of the second segment I_2 of the cycle $[T+1, 2T]$, to fill the capacity left free when the $\sigma(\beta)$ cells are exhausted.

We shall assume in the remaining part of this section that new calls are admitted into the network at the beginning of each segment of the partition $I = \{I_1, \ldots, I_q\}$ of each call cycle. For simplicity, we assume that all partition segments have the same number of steps, i.e. that

$$\lambda_j = \lambda, \qquad j = 1, \ldots, m$$

New calls are then admitted into the network at the times

$$0, T+1, T+\lambda+1, T+2\lambda+1, \ldots \tag{23}$$

and the traffic control algorithm takes the following form. Let $\sigma(*)$ denote the number of cells contained in the uncertain components stored during a call cycle. The cells $\sigma(*)$ stored during the first call cycle $[1, T]$ are released during the segment $[T+1, T+\lambda]$. The cells $\sigma(*)$ stored over the cycle $[T+1, 2T]$ are released during the segment $[2T+1, 2T+\lambda]$. The cells $\sigma(*)$ stored over the cycle $[T+\lambda+1, 2T+\lambda]$ are released over the segment $[2T+\lambda+1, 2T+2\lambda]$, and so on. This process continues indefinitely, and we refer to it as the *cycling process*.

So far, we have assumed that cell delays are not restricted. We remove now this assumption. In addition, we require that all cells stored during a call cycle be released during the first segment of the next call cycle. The latter restriction would be an inevitable requirement when the cell delay bounds satisfy

$$\tau(i) < 2\lambda, \qquad i = 1, \ldots, m$$

In order for it to be possible to release all stored cells during the first segment of the next call cycle, i.e. during the steps $T+1, \ldots, T+\lambda$, we must have $\sigma(\beta)/\lambda \leq \phi$, where ϕ is the backbone capacity. In view of (22), this is guaranteed whenever

$$\delta(\beta)/\lambda \leq \phi$$

When $\sigma(\beta)/\lambda < \phi$, the remaining empty backbone capacity is filled by admitting new calls at the step $T+1$. More new calls are then admitted later at the step $T+\lambda+1$, after the release of all stored cells $\sigma(\beta)$ has been completed.

The process of releasing the stored cells during the first segment of the next call cycle in effect extends the call cycle by one segment. To represent this extension, define a new call cycle time

$$T' := T + \lambda$$

On the interval $[1, T']$, induce the partition $I' := \{I_1, \ldots, I_{q+1}\}$ by adding the segment $I_{q+1} := [T+1, T+\lambda]$ to the original partition $I = \{I_1, \ldots, I_q\}$ of $[1, T]$. The original calls are then extended from $[1, T]$ to $[1, T']$ by defining them as zero over the segment I_{q+1}.

Now, define a rational function $\psi(\beta)$ over the interval $[1, T+\lambda]$ by setting

$$\left.\begin{aligned} &\psi_k(\beta) := \beta z, \qquad k = 1, \ldots, T \\ &\psi_k(\beta) = \sigma(\beta)/\lambda, \qquad k = T+1, \ldots, T+\lambda \end{aligned}\right\} \tag{24}$$

Here, z is the constant value of the package of deterministic parts (21), and $\sigma(\beta)$ is given by (22). Note that $\psi(\beta)$ is constant over each one of the portions $[1, T]$ and $[T+1, T+\lambda]$. The value of the function $\psi(\beta)$ over the segment I_{q+1} depends on the uncertain parts of the calls, and will vary from one sample of the call package to another.

To start our quantitative analysis, let $\mu(\cdot)$ denote the unit step function, i.e. $\mu(t) = 1$ when $t \geq 0$, and $\mu(t) = 0$ when $t < 0$. The quantity

$$\sigma(k, \beta) := \beta\rho \sum_{r=1}^{m} \alpha_r (k - \tau(r)) \mu(k - \tau(r)), \qquad k = 1, \ldots, T+\lambda$$

is the largest number of cells that might be in the buffers at the step k, and one can write

$$\sum_{r=1}^{m} \sum_{i=1}^{\min\{k-\tau(r), T\}} \alpha_r (\chi_i^r + \rho) = \sigma(k, \beta) + \sum_{r=1}^{m} \sum_{i=1}^{\min\{k-\tau(r), T\}} \alpha_r \chi_i^r$$

Since $0 \leq v^r \leq \rho$ for all $r = 1, \ldots, m$, we obtain

$$\sum_{r=1}^{m} \sum_{i=1}^{\min\{k-\tau(r), T\}} \alpha_r (\chi_i^r + v_i^r) \leq \sigma(k, \beta) + \sum_{r=1}^{m} \sum_{i=1}^{\min\{k-\tau(r), T\}} \alpha_r \chi_i^r, \quad k = 1, \ldots, T+\lambda$$

for all uncertainties $v^1, \ldots, v^m$. Now, define the special functions

$$\left.\begin{aligned} &\varphi_k(j, \beta) := \beta z, \qquad k = 1, \ldots, T \\ &\varphi_k(j, \beta) = \sigma(j, \beta)/\lambda, \qquad k = T+1, \ldots, T+\lambda \end{aligned}\right\} \tag{25}$$

$j = 1, \ldots, T+\lambda$. The following auxiliary result, whose proof is in the Appendix, brings us closer to the main statement of the present section.

Lemma 3: *The inequalities*

$$\sum_{r=1}^{m} \sum_{i=1}^{\min\{k-\tau(r), T\}} \alpha_r (\chi_i^r + v_i^r) \leq \sum_{i=1}^{k} \psi_i(\beta), \qquad k = 1, \ldots, T+\lambda \tag{26}$$

are valid for all uncertainties $0 \le v_i^r \le \rho$, $i = 1, \ldots, T$, $r = 1, \ldots, m$, *if and only if*

$$\sum_{r=1}^{m} \sum_{i=1}^{\min\{k-\tau(r),T\}} \alpha_r(\chi_i^r + \rho) \le \sum_{i=1}^{k} \varphi_i(k, \beta), \quad k = 1, \ldots, T + \lambda \qquad (27)$$

Recall from Hammer (2003, §4) that, for a complete family of call classes $F = \{\chi^1, \ldots, \chi^m\}$, the set of constancy coefficients $V^m(F)$ is the set of all integers $\beta_1, \ldots, \beta_m \ge 0$ for which the linear combination $\sum_{i=1}^m \beta_i \chi^i$ is constant and non-zero. The next statement provides necessary and sufficient conditions for achieving asymptotic efficiency of 1, despite call uncertainties.

Theorem 3: *Let* $c^i = \chi^i + v^i$, $i = 1, \ldots, m$, *be a family of call classes over the partition* $\{I_1, \ldots, I_q\}$ *of the interval* $[1, T]$, *where the uncertain parts satisfy* $0 \le v^i \le \rho$, $i = 1, \ldots, m$, $\rho > 0$. *Assume that the deterministic parts* $\chi^1, \ldots, \chi^m$ *form a complete family of call classes over* $[1, T]$. *Let* $\tau(i)$ *be the cell delay limitation of the class* c^i, $i = 1, \ldots, m$. *Assume that all partition segments* $I_1, \ldots, I_q$ *have the same number of steps* $\lambda \ge 1$, *and that* $\tau(i) < 2\lambda$, $i = 1, \ldots, m$. *Let* $\psi(1)$ *be the rational function of* (24) *for* $\beta = 1$. *Then, the following two statements are equivalent.*

(i) *There is a traffic control algorithm that achieves asymptotic efficiency of 1 for the family* $F = \{c^1, \ldots, c^m\}$.

(ii) *There is a set of integers* $\alpha_1, \ldots, \alpha_m \in V^m(\{\chi^1, \ldots, \chi^m\})$ *for which the following conditions hold:*

(a) $T\rho(\sum_{i=1}^m \alpha_i)/\lambda \le \sum_{i=1}^m \alpha_i \chi^i$, *and*

(b) $\sum_{r=1}^m \sum_{i=1}^{\min\{k-\tau(r),T\}} \alpha_r(\chi_i^r + \rho) \le \sum_{i=1}^k \varphi_i(k, 1)$, $k = 1, \ldots, T + \lambda$.

Proof: First note that the restrictions $\tau(i) < 2\lambda$, $i = 1, \ldots, m$, imply that all stored cells left over from the first call cycle $[1, T]$ must be transmitted during the segment $I_{q+1} = [T + 1, T + \lambda]$. Denote $T' := T + \lambda$. Assume now that part (i) of Theorem 3 is valid, and let ϕ be the backbone capacity. Let Ω denote the flow into the backbone over $[1, T']$. Then, by Hammer (2003, Theorem 1), Ω must be constant over the interval $[1, T]$. Furthermore, since all calls admitted at the step $T + 1$ are constant over the segment I_{q+1}, the same result implies that Ω must also be constant over I_{q+1}. Let Ω' be the constant value of Ω over $[1, T]$, and let Ω'' be its constant value over I_{q+1}. The values Ω' and Ω'' do not have to be equal, since new calls can be admitted at the step $T + 1$.

Due to the unpredictable nature of the uncertain parts $v^1, \ldots, v^m$, the backbone can be asymptotically filled over the interval $[1, T]$ only if the flow there is a combination of the deterministic parts $\chi^1, \ldots, \chi^m$. In view of Hammer (2003, Theorem 1), this implies that there are integers $\beta_1, \ldots, \beta_m \in V^m(\{\chi^1, \ldots, \chi^m\})$ such that $\Omega' = \sum_{i=1}^m \beta_i \chi_k^i$, $k = 1, \ldots, T$. Let $\beta > 0$ be an integer greatest common divisor of $\beta_1, \ldots, \beta_m$. Define integers $\alpha_1, \ldots, \alpha_m \ge 0$ by setting $\beta_i = \beta\alpha_i$, $i = 1, \ldots, m$, and note that $\alpha_1, \ldots, \alpha_m \in V^m(\{\chi^1, \ldots, \chi^m\})$. The number of stored cells is given by $\sigma(\beta) := \sum_{i=1}^m \sum_{k=1}^T \beta_i v_k^i$. Since all stored cells must be released during the interval I_{q+1}, it follows that $\sigma(\beta) = \lambda\Omega''$.

Extend now the call classes of the family F from the interval $[1, T]$ to the interval $[1, T + \lambda]$ by setting $\chi_k^i := 0$, $v_k^i := 0$, and $c^i := \chi^i + v^i = 0$ for all $k \in [T + 1, T + \lambda]$ and all $i = 1, \ldots, m$. We continue to use the symbol F to denote the resulting family of extended call classes. In view of the preceding, it must be possible to transform the incoming combination $z(c) := \beta \sum_{i=1}^m \alpha_i c^i$ over the interval $[1, T + \lambda]$ into the call class Ω over the time interval $[1, T + \lambda]$, for all possible samples of the uncertain parts $v^1, \ldots, v^m$. By Proposition 2, this implies that $\beta \sum_{r=1}^m \sum_{i=1}^{k-\tau(r)} \alpha_r c_i^r \le \sum_{i=1}^k \Omega_i$, $k = 1, \ldots, T + \lambda$. Since this condition must hold for all $0 \le v^i \le \rho$, condition (ii)(b) follows from Lemma 3, upon observing that

$$\varphi(k, 1) = \frac{1}{\beta}\varphi(k, \beta)$$

To show that (ii)(a) is also required, let

$$z := \sum_{i=1}^m \alpha_i \chi^i$$

and denote

$$\delta := T\rho\left(\sum_{i=1}^m \alpha_i\right) \Big/ \lambda - z$$

Assume, by contradiction, that $\delta > 0$, and consider the special case where the uncertainty samples satisfy $v^i = \rho$ for all $i = 1, \ldots, m$. Then, $\sigma(\beta) = \beta T\rho(\sum_{i=1}^m \alpha_i)$, and the flow amplitude Ω'' over I_{q+1} satisfies

$$\Omega'' = \sigma(\beta)/\lambda = \beta T\rho\left(\sum_{i=1}^m \alpha_i\right) \Big/ \lambda = \beta z + \beta\delta \qquad (28)$$

Now, in order to asymptotically fill the backbone over the interval $[1, T]$, we have to select β as the integer determined by the division algorithm $\phi = \beta z + \varpi$, where $0 \le \varpi < z$ (Hammer 2003, Proof of Theorem 1). Substituting into (28), we obtain $\Omega'' = \phi - \varpi + \beta\delta$. This yields $\Omega'' > \phi$ whenever $\beta > z/\delta$, a contradiction, since the capacity limit of the backbone is violated for sufficiently large values of β. Thus, we must have $\delta \le 0$, and condition (ii)(a) is valid. Together with the earlier part of the proof, this shows that (i) implies (ii).

Conversely, assume that condition (ii) of Theorem 3 holds. It follows then by Lemma 3 that inequalities (26) are valid for all possible uncertain parts. Let $\psi(1)$ be the function mentioned in (26). Since $\psi(1)$ is a rational function with a finite number of values, there is an integer $\beta > 0$ such that $\beta\psi(1)$ is an integer valued function. Define the call class

$$\Omega := \beta\psi(1)$$

Then, by Proposition 2, it is possible to transform the combination βz into the class Ω over the interval $[1, T+\lambda]$, for any values of the uncertain parts $0 \leq v^i \leq \rho$, $i = 1, \ldots, m$. Noting that Ω is constant over the intervals $[1, T]$ and I_{q+1}, it follows by the discussion preceding the present theorem that asymptotic efficiency of 1 can be achieved for the family F. This completes our proof. □

Note that the Shuffling Algorithm can be used to perform the waveform transformations used in the Proof of Theorem 3. The proof of Theorem 3 includes the following traffic control algorithm.

Traffic control of a complete family with uncertainties: Let $c^i = \chi^i + v^i$, $i = 1, \ldots, m$, be a family of call classes over the partition $\{I_1, \ldots, I_q\}$ of the interval $[1, T]$, where all partition segments have the same number of steps $\lambda \geq 1$. Assume that the deterministic parts $\{\chi^1, \ldots, \chi^m\}$ form a complete family of call classes, and that the conditions of Theorem 3(ii) are satisfied for the integers $\alpha_1, \ldots, \alpha_m$. Let $z = \sum_{i=1}^m \alpha_i \chi^i$ be the corresponding package of calls, and let ϕ be the backbone capacity. Then, for large backbone capacities ϕ, proceed as follows:

(i) Use the integer division algorithm to write $\phi = \beta_1 z + \varpi_1$, where $0 \leq \varpi_1 < z$. Admit β_1 packages at step 1.

(ii) Using the Shuffling Algorithm, transmit $\beta_1 z$ cells during each one of the steps $1, 2, \ldots, T$.

(iii) Let $\sigma(*)$ be the number of cells remaining stored in the buffers at the end of step T. Using the integer division algorithm, write $\sigma(*) = \gamma_1\lambda + \varepsilon_1$, where $0 \leq \varepsilon_1 < \lambda$. If $\varepsilon_1 = 0$, transmit into the backbone γ_1 stored cells during the steps $T+1, \ldots, T+\lambda$. If $\varepsilon_1 \neq 0$, transmit $\gamma_1 + 1$ stored cells during the steps $T+1, \ldots, T+\varepsilon_1$; during each of the steps $T+\varepsilon_1+1, \ldots, T+\lambda$, transmit γ_1 stored cells and one network control cell (to obtain constant amplitude). The Shuffling algorithm is used during these transmissions.

(iv) Denote $\phi_1 := \phi - (\gamma_1 + 1)$. Using the integer division algorithm, write $\phi_1 = \beta_2 z + \varpi_2$, where $0 \leq \varpi_2 < z$. Admit β_2 new call packages z at the step $T+1$.

(v) Using the shuffling algorithm, transmit $\beta_2 z$ cells of these calls during each one of the steps $T+1, T+2, \ldots, 2T$; store the remaining cells of these calls. The stored cells are transmitted during the segment $[2T+1, 2T+\lambda]$ as in step (iii).

(vi) Using the integer division algorithm, write $\gamma_1 = \beta_3 z + \varpi_3$, where $0 \leq \varpi_3 < z$. Admit β_3 call packages z at the step $T+\lambda+1$.

(vii) Using the shuffling algorithm, transmit $\beta_3 z$ cells of these calls during each one of the steps $T+\lambda+1, \ldots, 2T+\lambda$; store the remaining cells of these calls. These stored cells are transmitted during the segment $[2T+\lambda+1, 2T+2\lambda]$ as in step (iii).

(viii) Continue in this way, using the cycling process.

The execution of the Traffic Control Algorithm depends on the number of cells $\sigma(*)$ stored in the buffer system at the end of each call package. The number $\sigma(*)$ depends on the uncertain parts of the calls, and is not known in advance; it becomes known at the end of the call cycle through a feedback process. Consequently, the Traffic Control Algorithm depends critically on the use of feedback. This use of feedback, together with the shuffling algorithm, allows us to eliminate the effect of uncertainties on backbone efficiency, as long as the conditions of Theorem 3 are satisfied. It should be noted that the Traffic Control Algorithm does not cause oscillations in the backbone flow (compare to Schwartz 1996, Chapter 7, where possible oscillations in the backbone flow are discussed).

It is important to note that Theorem 3 and the Traffic Control Algorithm do not require a detailed statistical model of call uncertainties; only the uncertainty amplitude bound ρ is needed. Theorem 3 also shows that the possibility to eliminate the effect of uncertainties on the asymptotic efficiency of the backbone depends in a critical way on the values of the permissible cell delays $\tau(1), \ldots, \tau(m)$.

The conditions of Theorem 3 can be restated in the form of a limitation on the disturbance amplitude bound ρ, as follows. For a list of integers $\alpha_1, \ldots, \alpha_m \in V^m(\{\chi^1, \ldots, \chi^m\})$, define the quantities

$$\tau^* := \min\{\tau(r) \colon \alpha_r \neq 0, r = 1, \ldots, m\} \quad (29)$$

$$s(k) := (k - T)\sum_{i=1}^{m} \alpha_i(k - \tau(i))\mu(k - \tau(i))/\lambda, \qquad k = T+1, \ldots, T+\lambda \quad (30)$$

$$\varepsilon_k = \frac{\sum_{r=1}^{m}\sum_{i=\max\{1,\min[k-\tau(r),T]+1\}}^{k}\alpha_r\chi_i^r}{\sum_{r=1}^{m}(k-\tau(r))\mu(k-\tau(r))\alpha_r}$$

$$k=\tau^*+1,\ldots,T,\ \text{if } \tau^*+1\le T \quad (31)$$

$$\varepsilon_k = \frac{\sum_{r=1}^{m}\sum_{i=\max\{1,\min[k-\tau(r),T]+1\}}^{T}\alpha_r\chi_i^r}{[\sum_{r=1}^{m}(\min\{k-\tau(r),T\})\mu(\min\{k-\tau(r),T\})\alpha_r]-s(k)}$$

$$k=T+1,\ldots,T+\lambda \quad (32)$$

$$\varepsilon_{T+\lambda+1} := \frac{\lambda\sum_{i=1}^{m}\alpha_i\chi^i}{T(\sum_{i=1}^{m}\alpha_i)} \quad (33)$$

where, for any $k=1,\ldots,T+\lambda$, set $\varepsilon_k:=\infty$ if the denominator of the expression is less than or equal to zero for that value of k. Finally, define

$$\varepsilon(\alpha_1,\ldots,\alpha_m) := \min\{\varepsilon_k\colon k=1,\ldots,T+\lambda+1\} \quad (34)$$

Then, Theorem 3 can be restated in the following form, which constitutes the main result of the present section. It characterizes the largest uncertainty amplitude for which asymptotic efficiency of 1 can be obtained under the present circumstances.

Corollary 1: *Let $c^i=\chi^i+\upsilon^i$, $i=1,\ldots,m$, be a family of call classes over the partition $\{I_1,\ldots,I_q\}$ of the interval $[1,T]$, where all segments have the same number of steps $\lambda\ge 1$. Assume that $\chi^1,\ldots,\chi^m$ form a complete family of call classes, and that the uncertain parts satisfy $0\le\upsilon^i\le\rho$, $i=1,\ldots,m$. Let $\tau(i)$ be the maximal delay permissible for a cell of the class c^i, where $\tau(i)<2\lambda$, $i=1,\ldots,m$. Then, using (34), there is a traffic control algorithm that yields asymptotic efficiency of 1 if and only if the following is true. There is a list of integers $\alpha_1,\ldots,\alpha_m\in V^m(\{\chi^1,\ldots,\chi^m\})$ for which the uncertainty amplitude bound satisfies $\rho\le\varepsilon(\alpha_1,\ldots,\alpha_m)$.*

Proof: We show that the requirement $\rho\le\varepsilon(\alpha_1,\ldots,\alpha_m)$ is equivalent to conditions (ii) of Theorem 3. Recalling that $\psi_k(1)=\sum_{i=1}^{m}\alpha_i\chi_k^i$ for $k=1,\ldots,T$, and using the notation of (29)–(33), we can rewrite condition (ii)(b) of Theorem 3 in the form

$$\sum_{r=1}^{m}\sum_{i=1}^{\min\{k-\tau(r),T\}}\alpha_r\rho \le \sum_{r=1}^{m}\sum_{i=\max\{1,\min[k-\tau(r),T]+1\}}^{k}\alpha_r\chi_i^r$$

$$\text{for } k=1,\ldots,T \quad (35)$$

and

$$\sum_{r=1}^{m}\sum_{i=1}^{\min\{k-\tau(r),T\}}\alpha_r\rho \le \sum_{r=1}^{m}\sum_{i=\max\{1,\min[k-\tau(r),T]+1\}}^{T}\alpha_r\chi_i^r+\rho s(k)$$

$$\text{for } k=T+1,\ldots,T+\lambda \quad (36)$$

The last condition can be rewritten in the form

$$\rho\left[\left(\sum_{r=1}^{m}\sum_{i=1}^{\min\{k-\tau(r),T\}}\alpha_r\right)-s(k)\right] \le \sum_{r=1}^{m}\sum_{i=\max\{1,\min[k-\tau(r),T]+1\}}^{T}\alpha_r\chi_i^r$$

$$\text{for } k=T+1,\ldots,T+\lambda \quad (37)$$

Note that (36) is valid for any $\rho>0$ when the left side of (37) is negative. Expressing (35) and (37) as inequalities for ρ (while using the definition of the step function $\mu(\cdot)$) leads to the conclusion that condition (ii)(b) of Theorem 3 is equivalent to the requirement

$$\rho\le\min\{\varepsilon_k\colon k=1,\ldots,T+\lambda\}$$

Finally, in view of the definition of $\varepsilon_{T+\lambda+1}$, condition (ii)(a) of Theorem 3 is equivalent to the requirement $\rho\le\varepsilon_{T+\lambda+1}$. Thus, the requirement $\rho\le\min\{\varepsilon_k\colon k=1,\ldots,T+\lambda+1\}$ is equivalent to conditions (ii) of Theorem 3, and our proof concludes. □

We consider now several simple applications of Corollary 1. First, a slight reflection shows that the value of the denominator of (31, 32) cannot exceed $T(\sum_{i=1}^{m}\alpha_i)$. Also, define the quantity

$$c:=\min\{\chi_i^r\colon r=1,\ldots,m, i=1,\ldots,q\}$$

the minimal among the waveform values, and assume that $c\ne 0$. Assume also that there is at least one $\alpha_r\ne 0$ for which $\tau(r)>\lambda$. Then, the numerator of (31, 32) cannot be less than $c(\sum_{i=1}^{m}\alpha_i)$. In this case, we also have that $\sum_{i=1}^{m}\alpha_i\chi^i\le c(\sum_{i=1}^{m}\alpha_i)$, since c is a positive integer by assumption. Substituting these facts into (31–33), we conclude that, whenever there is at least one $\alpha_r\ne 0$ with $\tau(r)>\lambda$ and $c\ne 0$, one can achieve asymptotic efficiency of 1 if the disturbance amplitude satisfies

$$\rho\le\frac{c}{T}$$

This is, of course, just a simplified sufficient bound (larger values of ρ are possible; an accurate necessary and sufficient bound is provided by Corollary 1). Nevertheless, it provides insight into the nature of the traffic control problem when uncertainties are present.

Consider another set of simple special circumstances. Assume that the number of cells transferred by one of the classes, say χ^m, is much higher than the number of cells transferred by other classes. Let

$$\sigma^m:=\min\{\chi^m(1),\ldots,\chi^m(q)\}$$

Then, the numerator of (31, 32) is not less than $\alpha_m\sigma^m$, while the denominator is not more than $T(\sum_{i=1}^{m}\alpha_i)$, as before. Also, the numerator of (33) is not less than $\alpha_m \sigma^m$ for any value $\lambda \geq 1$. Thus, we obtain the bound

$$\rho \leq \frac{\alpha_m \sigma^m}{T(\sum_{i=1}^m \alpha_i)}$$

which is a convenient bound in cases where the bulk of the cells belong to the class χ^m. Whenever the uncertainty amplitude ρ does not exceed this bound, the backbone can be operated with asymptotic efficiency of 1, despite the uncertainties. Of couse, the bound given by Corollary 1 is, in general, larger.

It can be verified that $\rho \leq \varepsilon(\alpha_1, \ldots, \alpha_m)$ remains a sufficient condition for achieving asymptotic efficiency of 1 when the requirement $\tau(i) < 2\lambda$, $i = 1, \ldots, m$, is removed from Corollary 1. Consider then the special case when $\tau(r) > T + \lambda$ for all $r = 1, \ldots, m$, namely, when all call classes permit large delays. An examination shows that (31) does not apply in this case, since $\tau^* > T$, and that the denominator of (32) is zero. Consequently, $\varepsilon_k = \infty$ for all $k = 1, \ldots, T + \lambda$ in this case, and (33) is the only bound that needs to be enforced here. This leads to the following conclusion. If $\tau(r) > T + \lambda$ for all $r = 1, \ldots, m$, then asymptotic efficiency of 1 can be achieved whenever the uncertainty amplitude satisfies

$$\rho \leq \frac{\lambda \sum_{i=1}^m \alpha_i \chi^i}{T(\sum_{i=1}^m \alpha_i)}$$

As a final special case consider the following. Assume that one of the call classes, say the class c^m, has a permissible cell delay time $\tau(m) > T + \lambda$; that its population $\alpha_m > 0$; and that $\sigma^m > 0$. A direct examination of (31–33) shows that all conditions of Corollary 1 are satisfied when

$$\rho \leq \frac{\alpha_m \sigma^m}{T \sum_{r=1}^m \alpha_r} \tag{38}$$

Whence, in this case, asymptotic efficiency of 1 can be attained for all uncertainties that do not exceed this bound. Again, as in the earlier cases, the accurate bound of Corollary 1 will usually be larger than the current simplified sufficient bound.

In intuitive terms, the present special case can be viewed as follows. When the bound (38) is valid, the maximal total population of cells that originate from the uncertain parts of calls (i.e. $\rho T \sum_{r=1}^m \alpha_r$) does not exceed the number of deterministic cells arriving with calls of the class c^m. Since all cells of the class c^m can be delayed into the extension $[T+1, T+\lambda]$, the shuffling algorithm can transmit the uncertain parts of the other call classes right after their arrival, while delaying a corresponding number of cells of the class c^m to the extension $[T+1, T+\lambda]$.

We close with the following comment regarding buffer capacities.

Remark 1: The traffic control algorithms presented in the present paper require buffering of cells. Usually, most of the buffered cells can be stored in the buffer of the source controller C of (figure 1), rather than in the buffers of the router controller P. To be specific, let Δ be the propagation delay (in number of steps) between C and P. Then, it is not necessary to store in the buffers of P more than $2\Delta\phi$ cells, where ϕ is the backbone capacity.

Indeed, when a command is sent from P to C to summon more cells, it takes 2Δ steps for the new cells to arrive in P (Δ steps for the command to reach C and Δ steps for the new cells to reach P). Since the maximal number of cells that may exit P during the time interval 2Δ cannot exceed $2\Delta\phi$, no more than $2\Delta\phi$ cells need to be stored in P.

In most practical applications, the conduit represented by Σ in figure 1 is relatively short, so that Δ is relatively small. Consequently, the storage requirements imposed on the router controller P are relatively modest.

Appendix

Proof (of Proposition 1)**:** We start by constructing a special call class c which completes F into a complete family of calls $F(c)$. Let $z' := \sum_{i=1}^m \alpha_i^* c^i$ be an optimal flow achieving the maximal asymptotic efficiency η^* (Hammer 2003, Theorem 3), and let $A(z')$ be the amplitude of z'. Define a call class c^+ by setting

$$c^+(j) := A(z') - z'(j), \quad j = 1, \ldots, q \tag{39}$$

Then, the flow

$$z^+ := \sum_{i=1}^m \alpha_i^* c^i + c^+ \tag{40}$$

is clearly constant over $[1, T]$. Consequently, the augmented family $F(c^+) := \{c^1, \ldots, c^m, c^+\}$ is a complete family of calls by Hammer (2003, Theorem 1).

Next, let c be any call over the partition $\{I_1, \ldots, I_q\}$ for which the augmented family $F(c)$ is complete. For a backbone capacity ϕ, let $\alpha_1(\phi), \ldots, \alpha_{m+1}(\phi) \geq 0$ be a list of integers for which the flow $z(\phi) := \sum_{i=1}^m \alpha_i(\phi) c^i + \alpha_{m+1}(\phi) c$ is lossless and with asymptotic efficiency of 1. For this flow, the total number of cells of family F call classes transmitted by $z(\phi)$ is given by

$$n(F, z(\phi)) = \sum_{k=1}^T \sum_{i=1}^m \alpha_i(\phi) c_k^i$$

The ratio $n(F, z(\phi))/(T\phi)$ is, in fact, the efficiency of the flow $z(F, \phi) := \sum_{i=1}^m \alpha_i(\phi) c^i$ of the family F. Since η^* is the maximal asymptotic efficiency of the family F, it follows that $\lim_{\phi \to \infty} n(F, z(\phi))/(T\phi) \leq \eta^*$. Also, since

$z(\phi)$ is a lossless flow with asymptotic efficiency of 1, we have

$$\lim_{\phi\to\infty} \frac{\sum_{k=1}^{T} z_k(\phi)}{T\phi} = 1$$

Combining the last two sentences with the fact that

$$\eta(c, z, \phi) = \frac{\sum_{k=1}^{T} z_k(\phi) - n(F, z(\phi))}{T\phi}$$

we obtain

$$\lim_{\phi\to\infty} \eta(c, z, \phi) = \lim_{\phi\to\infty} \frac{\sum_{k=1}^{T} z_k(\phi) - n(F, z(\phi))}{T\phi} = 1 - \lim_{\phi\to\infty} \frac{n(F, z(\phi))}{T\phi} \geq 1 - \eta^* \quad (41)$$

This proves parts (i) and (ii) of Proposition 1.

Regarding part (iii) of Proposition 1, using the class c^+ of (39) and the integer division algorithm, let $\beta, \varepsilon \geq 0$ be the two integers satisfying $\phi = \beta A(z^+) + \varepsilon$, $0 \leq \varepsilon < A(z^+)$. Consider the flow $z(\phi) := \beta z^+$, where z^+ is the constant flow of (40). Then, $\sum_{k=1}^{T} z_k(\phi) = T\beta A(z^+)$ and $n(F, z(\phi)) = \sum_{k=1}^{T} \beta z'_k$. The construction of z' implies that $\lim_{\phi\to\infty} n(F, z(\phi))/(T\phi) = \eta^*$. Substituting into (41) with $c := c^+$, we get

$$\lim_{\phi\to\infty} \eta(c^+, z, \phi) = \lim_{\phi\to\infty} \frac{\sum_{k=1}^{T} z_k(\phi) - n(F, z(\phi))}{T\phi} = \lim_{\phi\to\infty} \left(\frac{T\beta A(z^+)}{T\phi} - \frac{n(F, z(\phi))}{T\phi} \right) = \lim_{\phi\to\infty} \left(\frac{\phi - \varepsilon}{\phi} - \frac{n(F, z(\phi))}{T\phi} \right) = 1 - \eta^*$$

where we have used the fact that $0 \leq \varepsilon < A(z^+)$. This completes our proof. □

Proof (of Lemma 3): Lemma 3 is a consequence of the following facts.

(i) For $k = 1, \ldots, T$, the right sides of (26) and (27) are identical.

(ii) For $k = 1, \ldots, T$, the right side of (26) is independent of the values of the uncertain parts. Whence the worst case of the inequalities (26) occurs when the uncertain parts included on the left side of (26) are at their maximum level ρ.

(iii) For $k = T + 1, \ldots, T + \lambda$, note that the inclusion of an extra cell in the uncertain part of the left side of (26) cannot increase the right side of (26) by more than one, since $\lambda \geq 1$. Thus, the worst case of the inequalities (26) for each $k = T + 1, \ldots, T + \lambda$ occurs when the uncertain parts included on the left side are at their maximal level ρ.

(iv) For $k = T + 1, \ldots, T + \lambda$, the lowest value of the right side of (26) occurs when the uncertain parts are zero at steps that are not included in the sum on the left side of (26). At the step k, such uncertain parts yield $\psi_i(\beta) = \varphi_i(k, \beta)$, $i = T + 1, \ldots, T + \lambda$. □

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