Proceedings of the Fifth International Conference of Applied Mathematics and Computing,
Plovdiv, Bulgaria, August 12-18, 2008

ROBUST OPTIMAL CONTROL: MAXIMUM TIME OF LOW-ERROR OPERATION

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The problem of maximizing the time during which a system operates below a specified error bound is considered for systems subject to perturbations and disturbances. It is shown that there is an optimal input function that keeps the response error below a specified magnitude for the longest period of time. The problem is formulated as a Max-Min optimization problem.

Specifically, consider a system $\Sigma(u, D, v, x_0, t)$ that has the input function $u(t)$; depends on parameters D; is affected by a disturbance signal $v(t)$; and has the initial condition x_0. Here, $D \in \Delta, v \in V$, and $x_0 \in \Xi$, where Δ, V, and Ξ are specified compact sets, and the input function u is taken from a set U of bounded Lebesgue measurable functions. Letting $\Sigma_0(t)$ be the desired nominal output signal, the response error is $\epsilon(u, D, v, x_0, t) := |\Sigma(u, D, v, x_0, t) - \Sigma_0(t)|$. The objective is to find an input function $u(t)$ that keeps the error $\epsilon(u, D, v, x_0, t)$ below a specified magnitude M for the longest time.

For given u, D, v, and x_0, the response error is acceptable up to the time $T(u, D, v, x_0) := \inf \{t \geq 0 : \epsilon(u, D, v, x_0, t) > M\}$. An optimal input function u^* maximizes this time for the worst perturbations and disturbances, yielding the overall best time

$$T^*(u^*) := \sup_{u \in U} \inf_{D \in \Delta, v \in V, x_0 \in \Xi} T(u, D, v, x_0).$$

It is shown that $u^*(t)$ exists under common conditions and that, without significantly affecting performance, u^* can be replaced by a bang-bang input function for easy calculation and implementation.

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1991 *Mathematics Subject Classification.* 49K35, 93C35.
Key words and phrases. Optimal Control, Max-Min Optimization, Bang-Bang Control.