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LINEAR SYSTEM FACTORIZATION: FEEDBACK AND STABILITY

by

Jacob Hammer† and Michael Heymann‡

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† Center for Mathematical System Theory, University of Florida, Gainesville, Florida 32611.

‡ Department of Electrical Engineering, Technion - Israel Institute of Technology, Haifa, Israel.

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Abstract

In HAUTUS and HEYMANN [1978], an investigation was initiated of the algebraic structure of discrete time, time invariant, finite dimensional linear systems (or, simply, linear systems) with particular emphasis on static state feedback. This investigation was extended to the study of dynamic as well as static output feedback in HAMMER and HEYMANN [1981]. Pivotal in the extended theory was the problem of causal factorization, i.e., the problem of factoring two system maps over each other through a causal factor. The theory was further extended in HAMMER and HEYMANN [1980] where the structural invariants of precompensation orbits and the concept of strict observability were studied in detail. Algebraically, the theory of strict observability hinges on the problem of polynomial factorization, i.e., the problem of factoring two system maps over each other through a polynomial factor.

It has since become increasingly clear, that the theory of linear systems can be formulated in a very general algebraic setup in which the central concepts of causality (and hence of feedback), of stability and of realization are investigated in a unified framework. In the present paper we present some of the essentials of this theory with particular emphasis on the issue of system stability. Some of the basic concepts on which the theory rests are reviewed briefly below.

Let K be a field, let S be a K-linear space and let ΛS denote the set of formal Laurent series in z^{-1} with coefficients in S, i.e., series of the form $s = \sum_{t=t_0}^{\infty} s_t z^{-t}$, $s_t \in S$. The set ΛK is a field under coefficientwise addition and convolutional multiplication and ΛS becomes a ΛK-linear space. Let ΩK be a principal ideal domain (P.I.D.) contained as a subring in ΛK. Then ΛS is an ΩK-module as well and so is also the set ΩS of all elements of the form $s = \Sigma \alpha_i s_i$, $\alpha_i \in \Omega K$, $s_i \in S$. If $\bar{f}: \Lambda U \to \Lambda Y$

is a ΛK-linear map, where U and Y are K-linear spaces (considered as the input and output value spaces, respectively, of a linear system) then $\bar{f}$ is simultaneously also an ΩK-homomorphism. This fact is of central importance in linear system theory.

Fundamental examples of PID's ΩK of interest in linear systems theory are the ring of polynomials $\Omega^{+}K$ which plays a basic role in realization theory, the ring of causal elements $\Omega^{-}K$ on which the theory of causal output feedback rests, and <u>stability rings</u> $\Omega_{\mathcal{D}}K$ of elements of the form p/q with $p \in \Omega^{+}K$ and $q \in \mathcal{D} \subset \Omega^{+}K$, where $\mathcal{D}$ is a properly defined <u>denominator set</u>. Since stability and causality are usually studied simultaneously (e.g., in connection with feedback), we are also interested in the ring $\Omega_{\mathcal{D}}^{-}K := \Omega_{\mathcal{D}}K \cap \Omega^{-}K$.

A ΛK-linear map $\bar{f}\colon \Lambda U \to \Lambda Y$ is called an ΩK-map if $\bar{f}[\Omega U] \subset \Omega Y$. Thus, we speak of a polynomial, causal, stable, stable and causal maps.

An important question in linear systems theory which can be regarded as a rather general problem of solvability and has many applications, is the problem of ΩK-factorization. That is, under what conditions does there exist for ΛK-linear maps $\bar{f}_1\colon \Lambda U \to \Lambda Y$ and $\bar{f}_2\colon \Lambda U \to \Lambda W$ an ΩK-map $\bar{h}\colon \Lambda Y \to \Lambda W$ such that $\bar{f}_2 = \bar{h}\bar{f}_1$. The basic theorem states that $\bar{h}$ exists if and only if $\mathrm{Ker}\,\pi_{\Omega K}\bar{f}_1 \subset \mathrm{Ker}\,\pi_{\Omega K}\bar{f}_2$, where for a ΛK-linear space ΛS, $\pi_{\Omega K}\colon \Lambda S \to \Lambda S/\Omega S$ is the canonical projection.

The ΩK-factorization theorem resolves in a unified way various well known problems of linear systems theory. In particular, it gives the basic realization theorems when applied to $\Omega^{+}K$, the causal output feedback problem when applied to $\Omega^{-}K$, and the causal output feedback problem with stability when applied to $\Omega_{\mathcal{D}}^{-}K$. Various other questions regarding stable feedback and compensation are investigated.

The set $\mathrm{Ker}\,\pi_{\Omega K}\bar{f}$ is an ΩK-submodule of ΛU. In fact, in case $\bar{f}$ is injective, it is a <u>bounded</u> ΩK-module. We study in detail the structure of bounded ΩK-modules, their <u>order</u> and <u>adapted bases</u>.

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