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CONTROLLING SEQUENTIAL MACHINES WITH DISTURBANCES

Jacob Hammer
Department of Electrical and Computer Engineering
University of Florida
Gainesville, FL 32611, USA
email: hammer@leaf.ee.ufl.edu

ABSTRACT

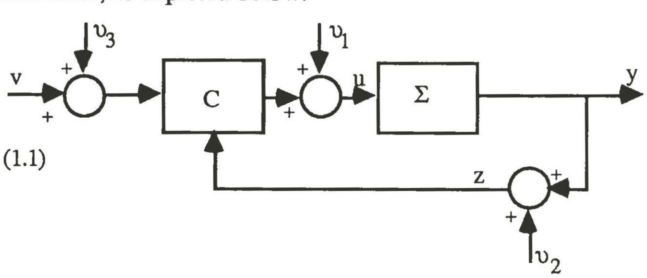
The problem of controlling a sequential machine under the influence of disturbances is considered. A methodology is developed for the design of controllers that guaranty that the effect of a "small" disturbance on the performance of the controlled machine remains "small". The methodology is based on a theory of fraction representations of sequential machines reminiscent of the general theory of fraction representations of nonlinear systems. This note is an extended summary of HAMMER [1996].

1. INTRODUCTION

Quite frequently one encounters the need to deal with sequential machines that are influenced by disturbances. The disturbances may originate from physical noise sources or from modeling uncertainties, or they may have a numerical origin. As an example of the former, consider a digital control system with remote telemetry. Here, noises in the telemetry communication channel create a disturbance that affects the system. As another example, consider a biochemical signaling chain in molecular biology (HAMMER [1995a and b]). Here, the natural random nature of biochemical processes can be viewed as a disturbance. Digital control systems and digital filtering systems furnish examples of sequential machines where disturbances of numerical origin may become important. Other examples of application areas abound.

Consider then a sequential machine Σ that operates within an environment with disturbances. To correct undesirable disturbance effects and improve the overall performance, connect Σ to another sequential machine C that serves as a controller, as depicted below.

(1.1)



Here, the composite system is influenced by three disturbances: an external input disturbance υ_3, an in-loop input disturbance υ_1, and an output disturbance υ_2. The only apriori information available about these disturbances is an amplitude bound, i.e., it is known that the amplitude of the disturbances υ_1, υ_2, and υ_3 cannot exceed a specified value. Other than that, no assumption is made as to the nature or the origin of the disturbances. The purpose of the controller C is to drive the system Σ so as to elicit from it desirable behavior, while accommodating the disturbances. As the figure indicates, the signal y is regarded as the output signal of the configuration. The external input signal is denoted by v. The symbol Σ_c will be used to indicate the input/output map induced by the closed loop system, so that, when the disturbances are absent, $y = \Sigma_c v$.

In the present paper we concentrate on the study of controllers C for which the effect of a disturbance on the output signal y does not exceed the original amplitude of the disturbance. We shall refer to such controllers as *disturbance attenuating* controllers. A disturbance attenuating controller guaranties that the disturbance is not amplified, so that "small" disturbances have only "small" effects on performance. The main result of the paper is the derivation of necessary and sufficient conditions for the existence of disturbance attenuating controllers, as well as the construction of such controllers, when they exist (for complete results, see HAMMER [1996]).

Following a long standing tradition in digital circuit theory and practice, we conduct our discussion within an input/output framework, where a sequential machine is considered as a system that maps input sequences of discrete values into output sequences of discrete values. Input/output representations are usually the most convenient form of specifying the desired characteristics of a system, and whence are the most common starting point for design considerations. The process of implementing a system involves the translation of the input/output description into a state representation, or realization, of the system (e.g., KOHAVI [1978]).

The effect of disturbances on closed loop systems is, of course, a central and widely studied subject in the literature on linear and nonlinear control theory. An important difference between the situation considered here and the standard literature is the fact that presently the systems operate over discrete spaces. Consequently, the standard notions of continuity and differentiability, which are commonly used to analyze the effects of small disturbances in classical control theory, need to be replaced with other appropriate notions (see sections 2 and 3 below).

An important aspect of the framework presented here is the development of a theory of fraction representations of sequential machines (section 3), in line with the general theory of fraction representations of nonlinear systems (HAMMER [1984a and b, 1985, 1994a], DESOER and KABULI [1988], VERMA [1988], VERMA and HUNT [1993], SONTAG

[1989], CHEN and de FIGUEIREDO [1990], PAICE and MOORE [1990], PAICE and van der SCHAFT [1994], BARAMOV and KIMURA [1995], and others). It turns out that fraction representations provide convenient tools for the investigation of disturbance attenuation for sequential machines, fulfilling here a role analogous to their role in the theory of control for systems over continuous spaces.

The sequential machines considered in this paper are recursive machines over the integers. To be specific, let Z be the set of integers, and let Z^n be the set of all n-dimensional vectors of integers. Then, we consider sequential machines Σ that permit a representation of the form

$$(1.2)\quad \begin{aligned} x_{k+1} &= f(x_k,u_k) \\ y_k &= h(x_k),\ k = 0, 1, 2, \dots \end{aligned}$$

where $x_k \in Z^n$ is an n-dimensional vector of integers called the *state* of Σ at the step k; $u_k \in Z^m$ is an m-dimensional vector of integers called the *input value* of Σ at the step k; and $y_k \in Z^p$ is a p-dimensional vector of integers called the *output value* of Σ at the step k. The function f is the *recursion function* of Σ, and h is the *output function*. We assume that an initial condition x_0 is provided, so that the response to an input sequence is uniquely determined.

Models of the form (1.2) are used to represent systems in many application areas. Some examples are models of biological signaling chains (HAMMER [1995a and b]); models of digital circuits used in computer hardware and software design; models of digital control systems; and models of digital filters.

In general terms, the material discussed in this paper is within the context of the theory of discrete-event systems, although the basic approach relies more heavily on concepts and techniques used in nonlinear control theory. The theory of discrete event systems offers a number of alternative treatments of problems related to the control of discrete systems; these include HOARE [1976], MILNER [1980], ARNOLD and NIVAT [1980], RAMADGE and WONHAM [1987], VAZ and WONHAM [1986], LIN and WONHAM [1988], THISTLE and WONHAM[1988], CIESLAK, DESCLAUX, FAWAZ, and VARAIYA [1988], OZVEREN and WILLSKY [1990], OZVEREN, WILLSKY, and ANTSAKLIS [1991]. Finally, the discussion of the present paper is a continuation of the work presented in HAMMER [1993, 1994b, c, 1995 a, b, and 1996].

2. BASIC CONCEPTS

2.1. Preliminaries.

We deal with sequential machines that operate on vectors of integers. Denote by Z the set of integers, and by Z^m the set of all m-dimensional vectors with integer entries. Let $S(Z^m)$ be the set of all sequences $(u_0, u_1, u_2, \dots)$ of m-dimensional integer vectors $u_k \in Z^m$, $k = 0, 1, 2, \dots$. Given a sequence $u \in S(Z^m)$, denote by u_k the k-th element of the sequence, where $k \geq 0$ is an integer. For an integer $k \geq 0$, we denote by u_0^k the list $u_0, u_1, \dots, u_k$.

A sequential machine that accepts sequences of m-dimensional integer vectors as input and generates sequences of p-dimensional integer vectors as output is simply a map $\Sigma : S(Z^m) \to S(Z^p)$. In many cases, the set of input sequences accepted by Σ is restricted; For instance, Σ may only permit input sequences whose amplitudes do not exceed a given bound. We denote by D_Σ the subset of $S(Z^m)$ that consists of all sequences that are allowed to serve as input sequences of the system Σ, and call D_Σ the *input domain* of Σ. In these terms, a sequential machine is represented by a map $\Sigma : D_\Sigma \to S(Z^p)$, where $D_\Sigma \subset S(Z^m)$ is the input domain of Σ. Our main interest is in sequential machines Σ that permit a recursive representation of the form (1.2).

Let u be an input sequence of a system Σ, and let $y := \Sigma u$ be the corresponding output sequence. We denote by $\Sigma u]_0^k$ the list of output values $y_0, y_1, \dots, y_k$.

A system $\Sigma : D_\Sigma \to S(Z^p)$ is *causal* (respectively, *strictly causal*) if the following holds for all input sequences $u, v \in D_\Sigma$: Whenever $u_0^k = v_0^k$ for some integer $k \geq 0$, then $\Sigma u]_0^k = \Sigma v]_0^k$ (respectively, $\Sigma u]_0^{k+1} = \Sigma v]_0^{k+1}$). It can be readily shown that a system with a recursive representation of the form (1.2) is strictly causal. A system $M : D_1 \to D_2$, where $D_1 \subset S(Z^m)$ and $D_2 \subset S(Z^p)$, is *bicausal* if it is a set isomorphism with both M and M^{-1} being causal.

A subset $\Delta \subset Z^m$ is called an *interval* if it is of the form $[a_1,b_1]\times\dots\times[a_m,b_m]$, where $b_i \geq a_i$, $i = 1, \dots, m$, are integers (and whence bounded). The interval $\Delta \subset Z^m$ induces an *interval* $S(\Delta)$ in $S(Z^m)$, which consists of all sequences in $S(Z^m)$ whose elements belong to Δ. Normally, the input domain of a sequential machine Σ is an interval in $S(Z^m)$.

A subset $D \subset S(Z^m)$ is *bounded* if it is contained in an interval. Similarly, a system $\Sigma : D_\Sigma \to S(Z^p)$ is *bounded* if its image $\mathrm{Im}\,\Sigma$ is contained within an interval in $S(Z^p)$.

In this paper the term *functional norm* refers to an assignment $|\cdot|$ that assigns a number $|f|$ to each function f of a given class, with the following properties: (a) For every function f and variables u and v, one has $|f(u) - f(v)| \leq |f||u - v|$; and (b) For every pair of functions f, g having appropriate domain and codomain, the composition gf satisfies $|gf| \leq |g||f|$. These properties make functional norms useful for the derivation of bounds on disturbance effects.

Our main subject involves the investigation of sequential machines that are influenced by perturbations and disturbances. To quantify the size of a disturbance or its effect, we shall use the standard ℓ^∞-norm.

Given a vector $v = (v^1, \dots, v^m) \in Z^m$, we denote by $|v| := \max\{|v^1|, \dots, |v^m|\}$ the largest absolute value of a component. For a sequence $u = (u_0, u_1, \dots) \in S(Z^m)$, let

$$|u| := \mathrm{Sup}_{i\geq 0}\, |u_i|$$

denote the usual ℓ^∞-norm.

2.2 Disturbance attenuating systems.

We consider now the effect of a disturbance of amplitude not exceeding δ, where $\delta > 0$ is an integer. A system is "disturbance attenuating" if any input disturbance of amplitude not exceeding δ causes an output deviation of amplitude not exceeding δ. In this sense, the system "attenuates" the class of disturbances of amplitude not exceeding δ. However, the output deviation caused by an input disturbance of amplitude less than δ may exceed the disturbance amplitude (but may not exceed δ). This broader sense of disturbance attenuation is satisfactory in many applications, and it broadens the class of systems for which disturbance attenuation can be achieved. The following notion is the basis of our discussion.

(For a positive real number $\alpha > 0$, denote by $[\alpha]^+$ the smallest integer that is not less than α.)

(2.1) DEFINITION. Let $g : \Delta_{in} \to Z^p$ be a function defined over the non empty domain $\Delta_{in} \subset Z^m$, and let $\delta > 0$ be an integer. The *δ-gain functional norm* $|g|_\delta$ of g is defined by the integer

$$|g|_\delta := \sup\left\{\left[\frac{|gu - gu'|}{d\delta}\right]^+ : d := \left[\frac{|u - u'|}{\delta}\right]^+ ; \begin{matrix} u,\ u' \in \Delta_{in} \\ u \neq u' \end{matrix}\right\}$$

The function g is *δ-attenuating* if $|g|_\delta \leq 1$. ♦

We adapt now the notion of δ-gain functional norm to causal systems. First, we define two projections for every integer $k \geq 0$. One is the projection

$p_k : S(Z^n) \to Z^n : p_k(u_0, u_1, \dots) := u_k$

that projects each sequence onto its k-th step; The second one is the projection

$P_k : S(Z^n) \to (Z^n)^{k+1} : P_k(u_0, u_1, \dots) := (u_0, \dots, u_k)$

that projects each sequence onto its initial $(k+1)$ elements. Now, consider a causal system $\Sigma : D_\Sigma \to S(Z^p)$ having the input domain $D_\Sigma \subset S(Z^m)$. Since Σ is causal, it follows that the output value $p_k\Sigma u$ is determined by the input values $P_k u$ for any input sequence $u \in S(Z^m)$. We can then define, for every integer $k \geq 0$, a function

(2.2) $\Sigma_{|k} : P_k D_\Sigma \to Z^p : \Sigma_{|k} P_k u := p_k \Sigma u$

defined for all points $(u_0, \dots, u_k) \in P_k D_\Sigma$ by the relation $\Sigma_{|k}(u_0, \dots, u_k) := p_k\Sigma u$, where $u \in D_\Sigma$ is a sequence for which $(u_0, \dots, u_k) = P_k u$. The family $\{\Sigma_{|k}\}$ characterizes the causal structure of the system Σ.

(2.3) DEFINITION. Let $\Sigma : D_\Sigma \to S(Z^p)$ be a causal system having the non-empty input domain $D_\Sigma \subset S(Z^m)$, and let $\delta > 0$ be an integer. For every integer $k \geq 0$, let $|\Sigma_{|k}|_\delta$ be the δ-gain functional norm of the function $\Sigma_{|k}$ of (2.2) induced by the system Σ. Then, the *δ-gain functional norm* $|\Sigma|_\delta$ of Σ is defined by

$$|\Sigma|_\delta := \sup_{k \geq 0} |\Sigma_{|k}|_\delta.$$

The system Σ is *δ-attenuating* if $|\Sigma|_\delta \leq 1$. ♦

It can be shown that the δ-gain functional norm fulfills the requirements for a functional norm (HAMMER [1996].

The construction of disturbance attenuating controllers depends in a critical way on our ability to extend the domain of functions without altering their δ-gain functional norm. This need leads us to an adaptation to our present discrete setup of an analog of the Tietze Lemma on the extension of continuous functions. To be specific, let $\Sigma : D_\Sigma \to D_o$ be a system having the bounded domain $D_\Sigma \subset S(Z^m)$. We show that, for any bounded domain $D \subset S(Z^m)$ containing D_Σ, there is an extension $\Sigma_e : D \to D_o$ of Σ whose δ-gain functional norm is equal to that of Σ. For notational simplicity, we use the symbol Σ for the extension Σ_e of Σ as well.

(2.4) PROPOSITION. Let $D_\Sigma \subset D \subset S(Z^m)$ be non empty bounded domains, let $D_o \subset S(Z^p)$ be an interval, let $\delta > 0$ be an integer, and let $\Sigma : D_\Sigma \to D_o$ be a causal system having the δ-gain functional norm $\gamma := |\Sigma|_\delta$. Then, there is a causal extension $\Sigma : D \to D_o$ of Σ whose δ-gain functional norm is still γ.

3. DISTURBANCE ATTENUATION

In the present section we derive necessary and sufficient conditions for the existence of a disturbance attenuating controller for a given sequential machine Σ. The construction of an appropriate controller is described in HAMMER [1996]. The discussion is based on a theory of fraction representations of sequential machines developed in this section.

We investigate the propagation of disturbances through the control configuration (1.1). Here, $\Sigma : D_\Sigma \to S(Z^p)$ is a given system, and C is a controller. Our main interest is in controllers C that guaranty disturbance attenuation for the closed loop system, where the term "disturbance attenuation" is discussed in detail below. In broad terms, disturbance attenuation means that the deviation caused by a disturbance of amplitude $\delta > 0$ does not exceed δ.

The input domain of the closed loop system is denoted by D_{in}, and it is required to be an interval in $S(Z^m)$. In this way, the closed loop system accepts any input sequence v whose element values stay within a prescribed range. For the sake of simplicity, we shall assume throughout our discussion that the system Σ is strictly causal and the controller C is causal. This guaranties that (1.1) is well posed. We use the notation

(3.1) $y = \Sigma_c(v, \upsilon_1, \upsilon_2, \upsilon_3)$

to denote the response of the closed loop system to the external signal v and the disturbances υ_1, υ_2, and υ_3. Since the configuration is well posed, the input signal u of Σ is uniquely determined by the external input signal v and the disturbances υ_1, υ_2, and υ_3, and we shall write

(3.2) $u = E(v, \upsilon_1, \upsilon_2, \upsilon_3)$,

where E is an appropriate system. It follows then directly that

$$\Sigma_c = \Sigma E.$$

We shall also use the notation $E_0(v) := E(v,0,0,0)$ and $\Sigma_{c0} := \Sigma_c(v,0,0,0)$ to indicate the noise-free response of the corresponding systems.

A frequent restriction on the controller C is that, for any output sequence y, the map $C(v,y)$ be an injective (one to one) function of the external input sequence v. A controller that satisfies this requirement is called a *reversible controller* (see HAMMER [1989a] for a more detailed discussion). For example, an additive feedback controller is always reversible. In intuitive terms, a reversible controller passes on to the controlled system Σ all the degrees of freedom available in the external input space D_{in}, so as to allow maximal utilization of the external input sequence to fine-tune the response of the closed loop system.

We turn now to our investigation of the effects of the disturbances υ_1, υ_2, and υ_3 on the control configuration (1.1). We shall require for each of these disturbances that whenever their amplitude is bounded by $\delta > 0$, the deviation they cause in *any* of the internal or external signals of the configuration also be bounded by δ. In this way we guaranty that the entire control configuration is not disturbed beyond a permissible bound. This requirement leads to the following notion of disturbance attenuation, which is in the spirit of the definition of internal stability used in nonlinear control theory.

(3.3) DEFINITION. Let $\delta > 0$ be an integer. The configuration (1.1) has a *disturbance attenuation radius* of δ if, for any disturbances $\upsilon_1, \upsilon_3 \in S(Z^m)$ and $\upsilon_2 \in S(Z^p)$ satisfying $|\upsilon_1| \leq \delta$, $|\upsilon_2| \leq \delta$, and $|\upsilon_3| \leq \delta$, and for any external input sequence v satisfying $v+\upsilon_3 \in D_{in}$, the following hold: (Here E is given by (3.2) and Σ_c is given by (3.1).)

(i) $|E(v+\upsilon_3,0,0) - E(v,0,0)| \leq \delta$,

(ii) $|E(v,\upsilon_1,0) - E(v,0,0)| \leq \delta$,

(iii) $|E(u,0,\upsilon_2) - E(u,0,0)| \leq \delta$;

(iv) $|\Sigma_c(v+\upsilon_3,0,0) - \Sigma_c(v,0,0)| \leq \delta$,

(v) $|\Sigma_c(v,\upsilon_1,0) - \Sigma_c(v,0,0)| \leq \delta$,

(vi) $|\Sigma_c(u,0,\upsilon_2) - \Sigma_c(u,0,0)| \leq \delta$. ♦

A disturbance attenuating control configuration gives rise to a particular fraction representation of the system Σ being controlled, as follows.

(3.4) PROPOSITION. Let $\Sigma : D_\Sigma \to S(Z^p)$ be a strictly causal system, with the input domain $D_\Sigma \subset S(Z^m)$. Assume there is a causal reversible controller $C : D_{in} \times S(Z^p) \to S(Z^m)$ for which the closed loop system (1.1) has disturbance attenuation radius $\delta > 0$. Then, the system Σ has a right fraction representation $\Sigma = ST^{-1}$, where $S : D_{in} \to S(Z^p)$ and $T : D_{in} \to \text{Im } T \subset S(Z^m)$ are causal δ-attenuating systems.

Proposition (3.4) is critical for the construction of disturbance attenuating controllers (HAMMER [1996]).

We continue now with our qualitative discussion of the effect of the disturbances υ_1, υ_2, and υ_3 on the configuration (1.1). So far, we have imposed the requirement that small disturbances cause only small deviations of the signals u and y. It is also important to address the question of whether or not it is possible to correct for these deviations, as small as they may be, through small changes in the external input sequence v. To be more specific, assume that the configuration has a disturbance attenuation radius $\delta > 0$, and consider, for example, a persistent (constant) disturbance υ_1 of amplitude not exceeding δ. Since the closed loop system has a disturbance attenuation radius of δ, the deviation of the signals u and y caused by this disturbance will not exceed δ. Nevertheless, a deviation has occurred. It would be natural to demand that it be possible to counteract this deviation (and return the signals u and y to their undisturbed values) by making a "small" adjustment to the external input signal v of the closed loop system. In broader terms, we shall require that it be possible to cancel the effect of any (known) disturbance signal of amplitude not exceeding $d\delta$, by a applying an adjustment of magnitude not exceeding $d\delta$ to the external input sequence v, where $d \geq 1$ is any integer. This leads to the following.

(3.5) DEFINITION. Let $\delta > 0$ be an integer. For a pair of input sequences u, u' of Σ in (1.1), denote by $y := \Sigma u$ and $y' := \Sigma u'$ the corresponding output sequences. Then, the configuration (1.1) with a reversible controller C is *strictly disturbance attenuating* with radius $\delta > 0$ if the following hold.

(i) The configuration has a disturbance attenuation radius of δ; and

(ii) Whenever $|u - u'| \leq d\delta$ and $|y - y'| \leq d\delta$ for some integer $d \geq 1$, there are external input sequences $v, v' \in D_{in}$ for which $u = E_0 v$, $u' = E_0 v'$, and $|v - v'| \leq d\delta$. ♦

We briefly interrupt our examination of disturbance attenuation in order to review the notion of a graph.

Let $\Sigma : D_\Sigma \to S(Z^p) : u \mapsto \Sigma u$ be a strictly causal system with the non-empty input domain $D_\Sigma \subset S(Z^m)$. As usual, the *graph* G_Σ of Σ is a subset of the cross product space $S(Z^p) \times S(Z^m)$, consisting of all pairs $(\Sigma u, u)$, $u \in D_\Sigma$. The graph of the system Σ plays an important role in our discussion, compatible with its role in the general theory of nonlinear control systems over topological spaces (HAMMER [1984a], [1985]). Proposition (3.4) has certain implications on the structure of the graph of Σ, as we discuss next.

First, we need some terminology. A system M for which M and M^{-1} are both δ-attenuating is called a δ-*unimodular* system, or a δ-*homeomorphism.* A subset $\Gamma \subset G_\Sigma$ is δ-*homeomorphic* to a subset $D \subset S(Z^m)$ if there is a bicausal and δ-unimodular set isomorphism $M : D \to \Gamma$.

Assume there is a causal reversible controller $C : D_{in} \times S(Z^p) \to S(Z^m)$, where $D_{in} \subset S(Z^m)$ is an interval, for which the control configuration (1.1) around the given system Σ has a disturbance attenuation radius $\delta > 0$. Then, as Proposition 3.4 indicates, there is a right fraction representation $\Sigma = ST^{-1}$, where $S : D_{in} \to S(Z^p)$ and $T : D_{in} \to \text{Im } T$ are δ-attenuating causal systems. For every sequence

$v \in D_{in}$, the sequences $u := Tv$ and $y := Sv$ satisfy $y = Sv = (ST^{-1})Tv = \Sigma Tv = \Sigma u$, so that the pair $(Sv,Tv) = (\Sigma u,u)$ is a point of the graph of Σ. Consequently, the set

$$\Gamma = \{(Sv,Tv),\ v \in D_{in}\}$$

is a subset of the graph G_Σ of Σ. Define the map

$$M := D_{in} \to \Gamma : Mv := (Sv,Tv).$$

We claim that M is a set isomorphism. Indeed, M is surjective (onto) by the definition of the set Γ, and it is injective since T is injective. Furthermore, the fact that S and T are both δ-attenuating causal systems implies that M is a δ-attenuating causal system as well. Taking into account strict disturbance attenuation, it can be shown that M is in fact a δ-unimodular system. Furthermore, the converse of this fact is also true, namely, if the graph of Σ contains a subset that is δ-homeomorphic to an interval, then there is a controller for Σ that provides strict disturbance attenuation, as follows.

(3.6) THEOREM. Let $\Sigma : D_\Sigma \to S(Z^p)$ be a strictly causal system with the non-empty input domain $D_\Sigma \subset S(Z^m)$, and let $\delta > 0$ be an integer. Assume there is a bounded subset Γ of the graph of Σ that is δ-homeomorphic to an interval $D_{in} \subset S(Z^m)$. Then, there is a causal reversible controller C for which the configuration (1.1) around Σ is strictly disturbance attenuating with radius δ, and has the external input domain D_{in}.

Moreover, the δ-homeomorphism of Theorem 3.6 can be directly used to construct a controller that yields strict disturbance attenuation for the system Σ (see HAMMER [1996]).Thus, the existence of a subset of the graph of Σ that is homeomorphic to an interval is a necessary and sufficient condition for strict disturbance attenuation. This fact is closely analogous to the situation encountered in the theory of robust stabilization of nonlinear systems over topological spaces (HAMMER [1989b]), and can be viewed as a general principle of control theory.

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